Computer Algebra
Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/149-5.3.3-d+e-x-^m-a+b-arctan-c-x^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [31]. This is test number [149].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (31)	0.00 (0)
Maple	96.77 (30)	3.23 (1)
Mathematica	87.10 (27)	12.90 (4)
Mupad	45.16 (14)	54.84 (17)
Maxima	45.16 (14)	54.84 (17)
Fricas	38.71 (12)	61.29 (19)
Sympy	35.48 (11)	64.52 (20)
Giac	19.35 (6)	80.65 (25)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

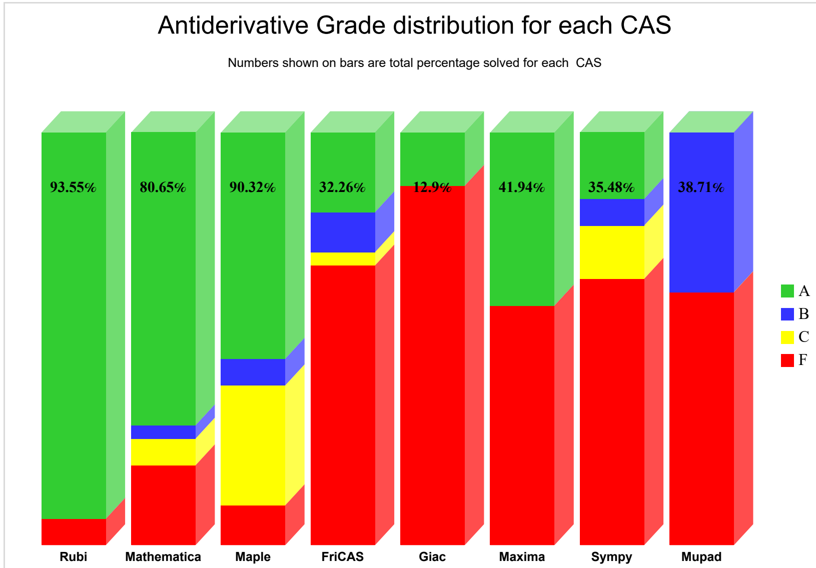
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

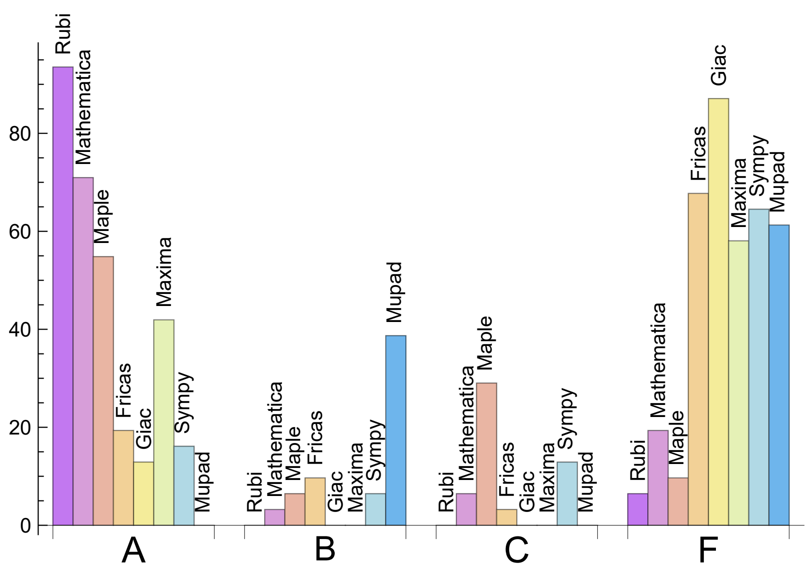
System	% A grade	% B grade	% C grade	% F grade
Rubi	93.548	0.000	0.000	6.452
Mathematica	70.968	3.226	6.452	19.355
Maple	54.839	6.452	29.032	9.677
Maxima	41.935	0.000	0.000	58.065
Fricas	19.355	9.677	3.226	67.742
Sympy	16.129	6.452	12.903	64.516
Giac	12.903	0.000	0.000	87.097
Mupad	0.000	38.710	0.000	61.290

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Mathematica	4	75.00	25.00	0.00
Maxima	17	82.35	11.76	5.88
Mupad	17	0.00	100.00	0.00
Sympy	20	60.00	40.00	0.00
Fricas	19	84.21	5.26	10.53
Giac	25	88.00	12.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.31
Rubi	0.35
Mupad	1.50
Giac	2.41
Sympy	5.86
Fricas	6.49
Mathematica	13.67
Maple	15.04

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	180.00	1.03	210.00	1.05
Maxima	223.64	1.20	223.00	1.16
Rubi	352.74	1.00	270.00	1.00
Mupad	463.86	1.78	238.00	1.43
Mathematica	464.67	1.19	297.00	1.10
Sympy	1409.45	7.86	345.00	1.82
Maple	2087.07	3.76	300.00	1.27
Fricas	206765.25	631.35	230.00	1.62

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

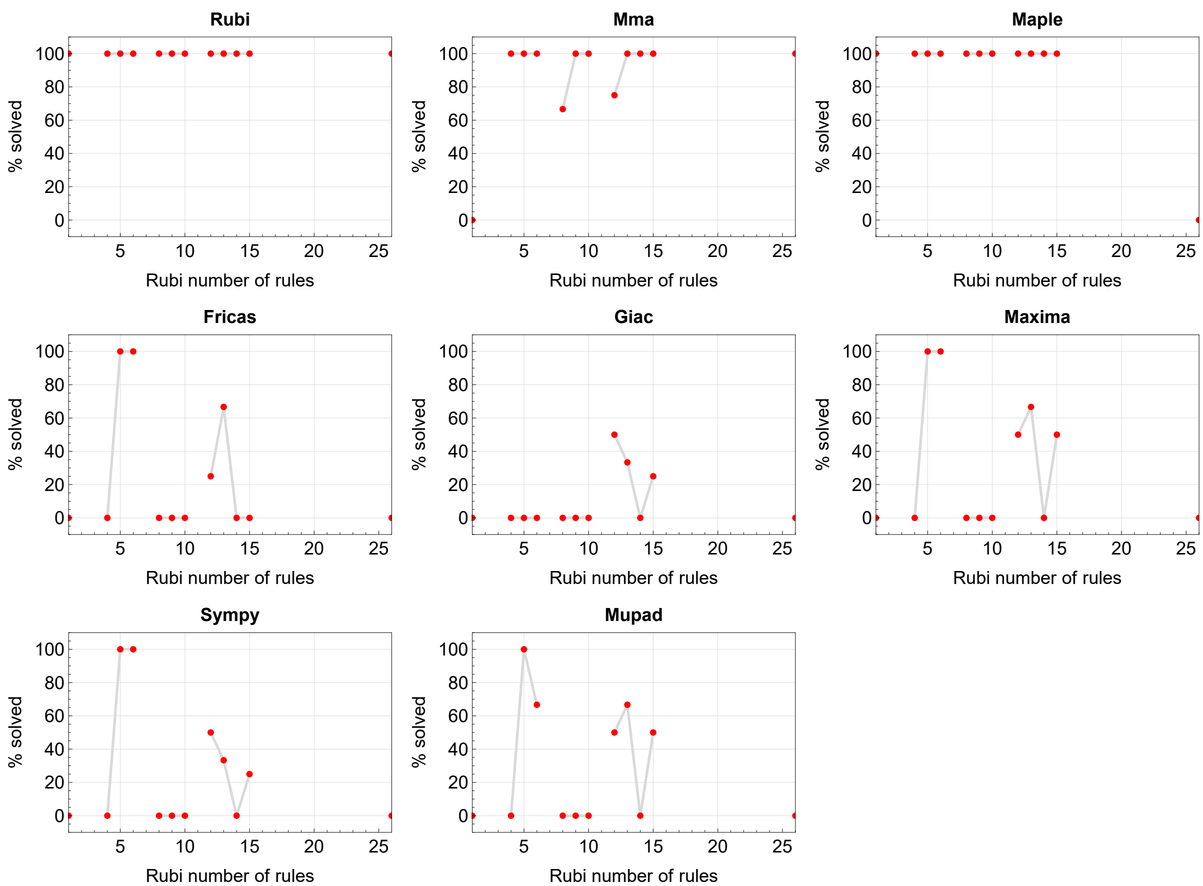


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

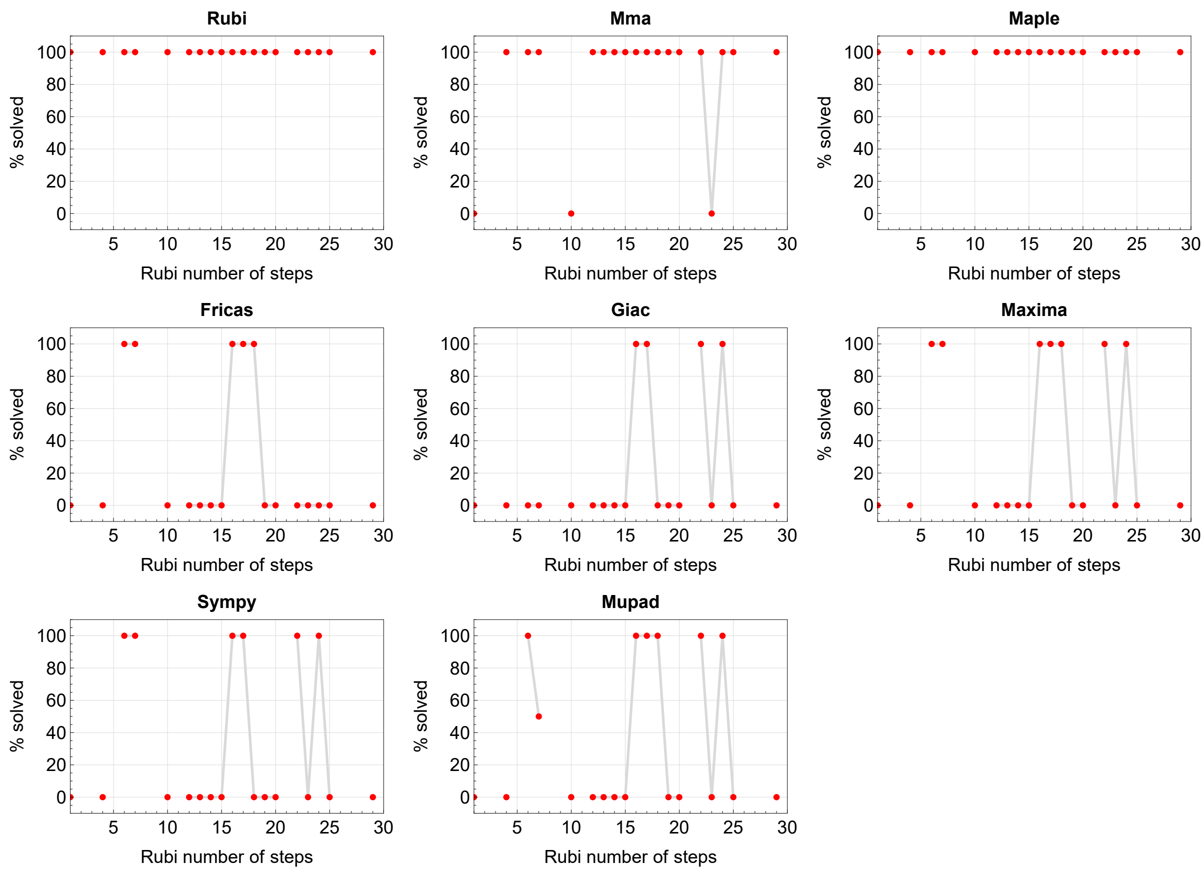


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

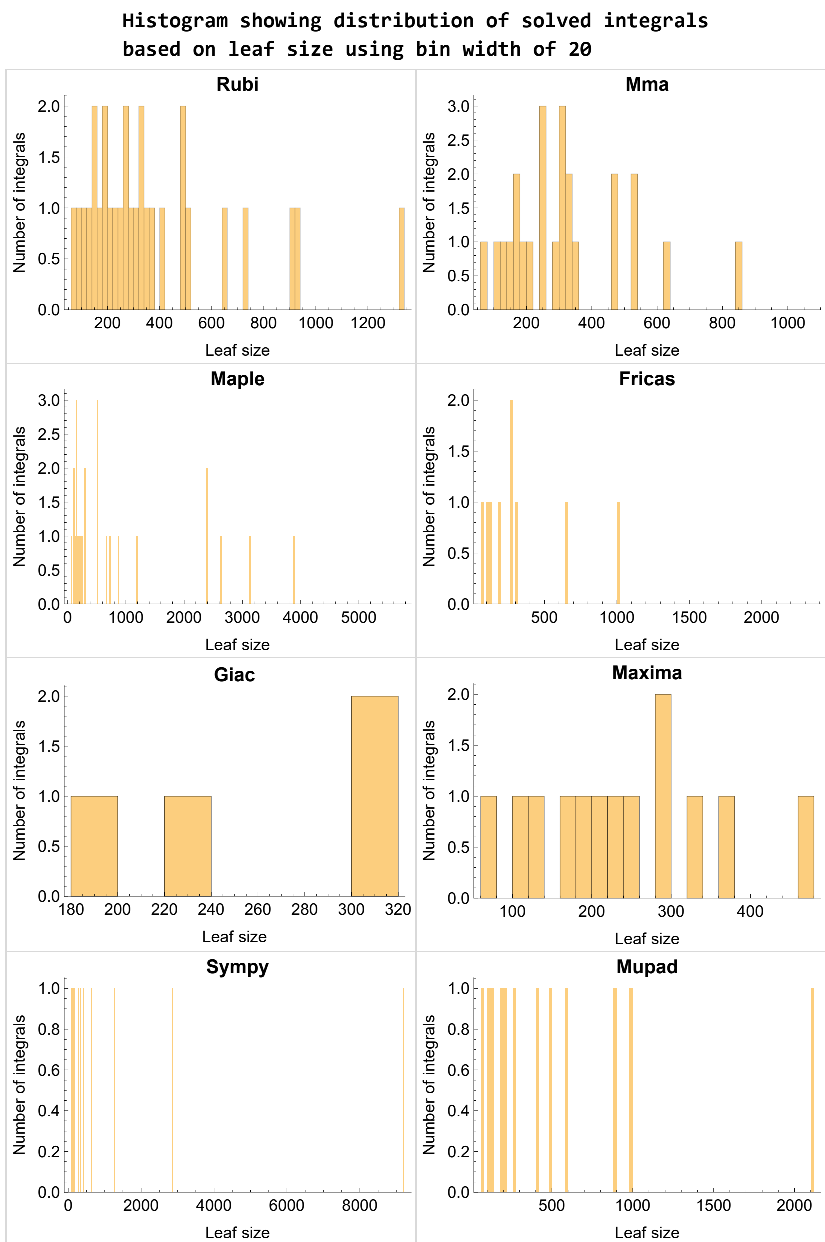


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

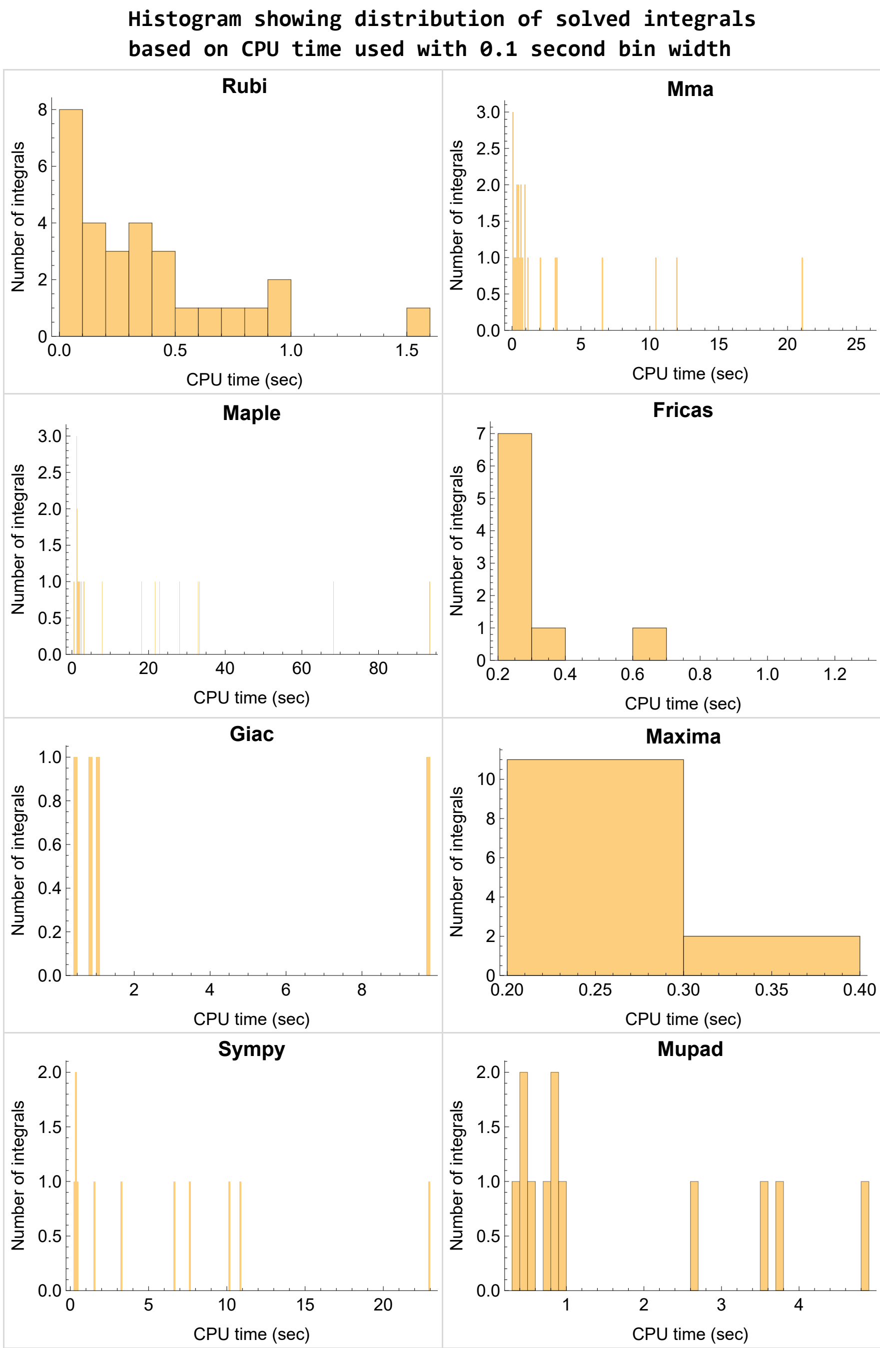


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

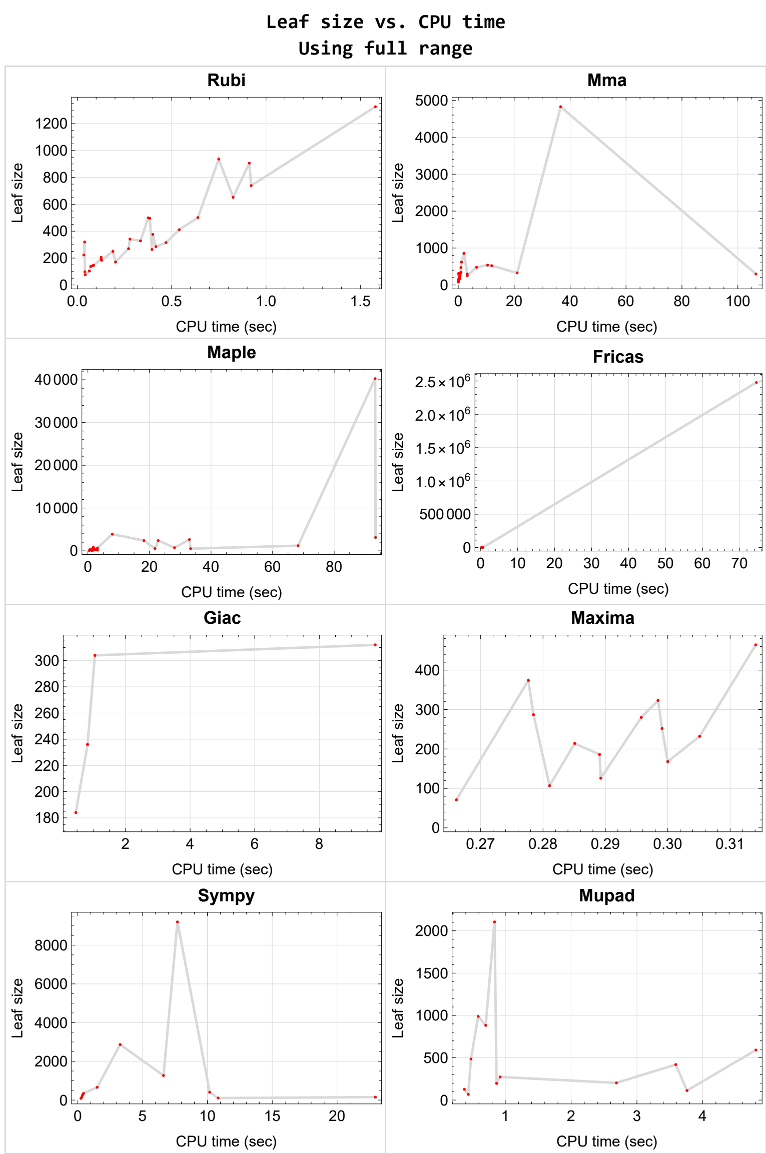


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{26, 27\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{25\}$

Maple $\{12, 15, 16, 17, 18, 19, 20\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

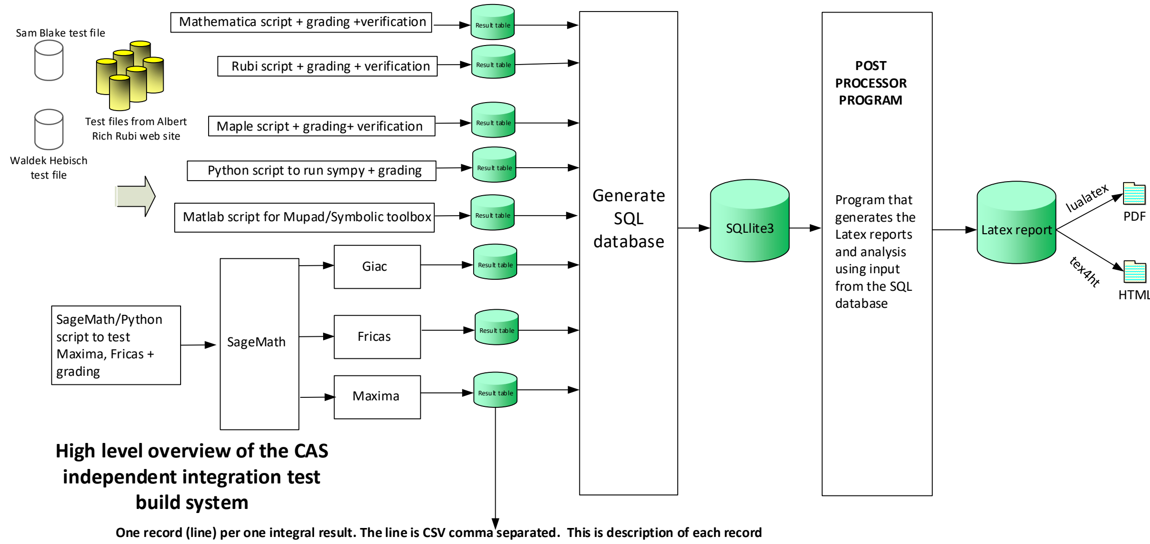
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 21, 22, 24, 28, 29, 31 }

B grade { 25 }

C grade { 23, 30 }

F normal fail { 12, 19, 20 }

F(-1) timedout fail { 18 }

F(-2) exception fail { }

Maple

A grade $\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 21, 22, 24, 29, 31 \}$

B grade $\{ 10, 28 \}$

C grade $\{ 12, 15, 16, 17, 18, 19, 20, 23, 30 \}$

F normal fail $\{ 25 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ \}$

Fricas

A grade $\{ 1, 2, 3, 4, 6, 22 \}$

B grade $\{ 7, 8, 21 \}$

C grade $\{ 24 \}$

F normal fail $\{ 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 25, 30 \}$

F(-1) timedout fail $\{ 31 \}$

F(-2) exception fail $\{ 28, 29 \}$

Maxima

A grade $\{ 1, 2, 3, 4, 6, 7, 8, 21, 22, 24, 28, 29, 31 \}$

B grade $\{ \}$

C grade $\{ \}$

F normal fail $\{ 5, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 23, 25, 30 \}$

F(-1) timedout fail $\{ 14, 20 \}$

F(-2) exception fail $\{ 27 \}$

Giac

A grade $\{ 21, 22, 28, 29 \}$

B grade $\{ \}$

C grade $\{ \}$

F normal fail $\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 23, 24, 25, 30, 31 \}$

F(-1) timedout fail $\{ 18, 19, 20 \}$

F(-2) exception fail $\{ \}$

Mupad

A grade $\{ \}$

B grade $\{ 1, 2, 3, 4, 6, 7, 21, 22, 24, 28, 29, 31 \}$

C grade $\{ \}$

F normal fail $\{ \}$

F(-1) timedout fail $\{ 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 25, 30 \}$

F(-2) exception fail $\{ \}$

Sympy

A grade $\{ 3, 4, 21, 28, 29 \}$

B grade $\{ 1, 2 \}$

C grade $\{ 6, 7, 8, 22 \}$

F normal fail $\{ 5, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 25 \}$

F(-1) timedout fail $\{ 14, 20, 23, 24, 26, 27, 30, 31 \}$

F(-2) exception fail $\{ \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	255	246	252	264	345	0	273
N.S.	1	1.00	1.39	1.34	1.37	1.43	1.88	0.00	1.48
time (sec)	N/A	0.129	0.516	2.574	0.299	0.255	0.451	0.000	0.922

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	218	191	186	196	262	0	197
N.S.	1	1.00	1.51	1.33	1.29	1.36	1.82	0.00	1.37
time (sec)	N/A	0.083	0.475	1.485	0.289	0.257	0.371	0.000	0.868

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	163	119	126	133	160	0	127
N.S.	1	1.00	1.58	1.16	1.22	1.29	1.55	0.00	1.23
time (sec)	N/A	0.064	0.395	1.224	0.289	0.247	0.323	0.000	0.378

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	69	71	71	87	0	67
N.S.	1	1.00	1.01	0.91	0.93	0.93	1.14	0.00	0.88
time (sec)	N/A	0.042	0.009	0.418	0.266	0.250	0.249	0.000	0.438

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	138	156	0	0	0	0	0
N.S.	1	1.00	1.00	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.076	3.097	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	111	110	107	116	658	0	112
N.S.	1	1.00	1.13	1.12	1.09	1.18	6.71	0.00	1.14
time (sec)	N/A	0.040	0.232	1.576	0.281	0.268	1.503	0.000	3.760

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	192	153	214	313	2866	0	591
N.S.	1	1.00	1.32	1.05	1.47	2.14	19.63	0.00	4.05
time (sec)	N/A	0.086	0.356	1.623	0.285	0.346	3.271	0.000	4.808

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	254	204	374	642	9202	0	0
N.S.	1	1.00	1.23	0.99	1.82	3.12	44.67	0.00	0.00
time (sec)	N/A	0.127	0.678	1.947	0.278	0.639	7.693	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	472	667	0	0	0	0	0
N.S.	1	1.00	1.26	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	0.951	3.124	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	312	503	0	0	0	0	0
N.S.	1	1.00	1.16	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.605	2.078	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	172	292	0	0	0	0	0
N.S.	1	1.00	1.01	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.482	1.333	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	223	223	0	1199	0	0	0	0	0
N.S.	1	1.00	0.00	5.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.000	68.260	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	300	513	0	0	0	0	0
N.S.	1	1.00	0.88	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	3.199	21.774	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	479	729	0	0	0	0	0
N.S.	1	1.00	0.97	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	6.540	28.132	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	652	652	855	3122	0	0	0	0	0
N.S.	1	1.00	1.31	4.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.826	2.007	93.443	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	411	411	621	2633	0	0	0	0	0
N.S.	1	1.00	1.51	6.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.540	1.163	32.974	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	264	264	342	3886	0	0	0	0	0
N.S.	1	1.00	1.30	14.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	0.978	7.927	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	320	320	0	2398	0	0	0	0	0
N.S.	1	1.00	0.00	7.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.000	18.202	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	499	499	0	2398	0	0	0	0	0
N.S.	1	1.00	0.00	4.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.000	22.853	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	936	936	0	40258	0	0	0	0	0
N.S.	1	1.00	0.00	43.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.750	0.000	93.310	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	252	303	323	1013	403	304	419
N.S.	1	1.00	1.01	1.21	1.29	4.05	1.61	1.22	1.68
time (sec)	N/A	0.189	3.220	2.214	0.298	0.278	10.176	1.052	3.590

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	153	146	168	274	1266	184	203
N.S.	1	1.00	0.80	0.76	0.88	1.43	6.59	0.96	1.06
time (sec)	N/A	0.126	0.112	0.569	0.300	0.256	6.616	0.461	2.689

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	326	138	0	0	0	0	0
N.S.	1	1.00	0.65	0.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.639	21.058	1.266	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	321	297	287	2478078	0	0	883
N.S.	1	1.00	0.98	0.91	0.88	7555.12	0.00	0.00	2.69
time (sec)	N/A	0.335	0.756	1.395	0.278	74.552	0.000	0.000	0.702

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1325	1325	4824	0	0	0	0	0	0
N.S.	1	1.00	3.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.579	36.630	0.000	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	47	36	0	22	22
N.S.	1	1.00	1.10	1.00	2.35	1.80	0.00	1.10	1.10
time (sec)	N/A	0.020	51.612	0.450	0.618	0.230	0.000	0.280	0.385

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	0	47	0	22	22
N.S.	1	1.00	1.10	1.00	0.00	2.35	0.00	1.10	1.10
time (sec)	N/A	0.019	108.058	1.016	0.000	0.247	0.000	2.098	0.491

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	297	503	280	0	151	312	988
N.S.	1	1.00	0.94	1.60	0.89	0.00	0.48	0.99	3.14
time (sec)	N/A	0.470	106.475	33.358	0.296	0.000	22.953	9.729	0.588

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	310	305	232	0	104	236	485
N.S.	1	1.00	1.09	1.07	0.81	0.00	0.36	0.83	1.70
time (sec)	N/A	0.416	0.076	0.698	0.305	0.000	10.826	0.825	0.478

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	739	739	522	172	0	0	0	0	0
N.S.	1	1.00	0.71	0.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.922	11.976	1.235	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	906	906	536	862	464	0	0	0	2105
N.S.	1	1.00	0.59	0.95	0.51	0.00	0.00	0.00	2.32
time (sec)	N/A	0.911	10.475	1.727	0.314	0.000	0.000	0.000	0.836

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [25] had the largest ratio of [1.4439999999999995]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	16	0.312
2	A	6	5	1.00	16	0.312
3	A	6	5	1.00	16	0.312
4	A	6	5	1.00	14	0.357
5	A	4	4	1.00	16	0.250
6	A	6	6	1.00	16	0.375
7	A	7	6	1.00	16	0.375
8	A	7	6	1.00	16	0.375
9	A	19	14	1.00	18	0.778
10	A	15	12	1.00	18	0.667
11	A	12	9	1.00	16	0.562
12	A	1	1	1.00	18	0.056
13	A	13	9	1.00	18	0.500
14	A	19	15	1.00	18	0.833
15	A	29	15	1.00	18	0.833
16	A	20	13	1.00	18	0.722
17	A	14	10	1.00	16	0.625
18	A	1	1	1.00	18	0.056
19	A	10	8	1.00	18	0.444
20	A	23	12	1.00	18	0.667
21	A	17	13	1.00	18	0.722
22	A	16	12	1.00	16	0.750
23	A	19	8	1.00	18	0.444
24	A	18	13	1.00	18	0.722

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	77	26	1.00	18	1.444
26	N/A	0	0	1.00	20	0.000
27	N/A	0	0	1.00	20	0.000
28	A	24	15	1.00	18	0.833
29	A	22	12	1.00	16	0.750
30	A	25	8	1.00	18	0.444
31	A	34	15	1.00	18	0.833

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (d+ex)^4(a+b\arctan(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 184

$$\begin{aligned}\int (d+ex)^4(a+b\arctan(cx))\,dx = &-\frac{bde(2c^2d^2-e^2)\,x}{c^3}-\frac{be^2(10c^2d^2-e^2)\,x^2}{10c^3}-\frac{bde^3x^3}{3c}\\&-\frac{be^4x^4}{20c}-\frac{bd(c^4d^4-10c^2d^2e^2+5e^4)\arctan(cx)}{5c^4e}\\&+\frac{(d+ex)^5(a+b\arctan(cx))}{5e}\\&-\frac{b(5c^4d^4-10c^2d^2e^2+e^4)\log(1+c^2x^2)}{10c^5}\end{aligned}$$

```
[Out] -b*d*e*(2*c^2*d^2-e^2)*x/c^3-1/10*b*e^2*(10*c^2*d^2-e^2)*x^2/c^3-1/3*b*d*e^
3*x^3/c-1/20*b*e^4*x^4/c-1/5*b*d*(c^4*d^4-10*c^2*d^2*e^2+5*e^4)*arctan(c*x)
/c^4/e+1/5*(e*x+d)^5*(a+b*arctan(c*x))/e-1/10*b*(5*c^4*d^4-10*c^2*d^2*e^2+e
^4)*ln(c^2*x^2+1)/c^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4972, 716, 649, 209, 266}

$$\int (d+ex)^4(a+b\arctan(cx))\,dx = \frac{(d+ex)^5(a+b\arctan(cx))}{5e} - \frac{bd\arctan(cx)\left(c^4d^4-10c^2d^2e^2+5e^4\right)}{5c^4e} - \frac{be^2x^2(10c^2d^2-e^2)}{10c^3} - \frac{bdex(2c^2d^2-e^2)}{c^3} - \frac{b(5c^4d^4-10c^2d^2e^2+e^4)\log\left(c^2x^2+1\right)}{10c^5} - \frac{bde^3x^3}{3c} - \frac{be^4x^4}{20c}$$

```
[In] Int[(d + e*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*d*e*(2*c^2*d^2 - e^2)*x)/c^3) - (b*e^2*(10*c^2*d^2 - e^2)*x^2)/(10*c^3
) - (b*d*e^3*x^3)/(3*c) - (b*e^4*x^4)/(20*c) - (b*d*(c^4*d^4 - 10*c^2*d^2*e
^2 + 5*e^4)*ArcTan[c*x])/(5*c^4*e) + ((d + e*x)^5*(a + b*ArcTan[c*x]))/(5*e
) - (b*(5*c^4*d^4 - 10*c^2*d^2*e^2 + e^4)*Log[1 + c^2*x^2])/(10*c^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^5(a+b\arctan(cx))}{5e} - \frac{(bc)\int \frac{(d+ex)^5}{1+c^2x^2}\,dx}{5e} \\
&= \frac{(d+ex)^5(a+b\arctan(cx))}{5e} \\
&\quad - \frac{(bc)\int \left(\frac{5de^2(2c^2d^2-e^2)}{c^4} + \frac{e^3(10c^2d^2-e^2)x}{c^4} + \frac{5de^4x^2}{c^2} + \frac{e^5x^3}{c^2} + \frac{c^4d^5-10c^2d^3e^2+5de^4+e(5c^4d^4-10c^2d^2e^2+e^4)x}{c^4(1+c^2x^2)}\right)dx}{5e} \\
&= -\frac{bde(2c^2d^2-e^2)x}{c^3} - \frac{be^2(10c^2d^2-e^2)x^2}{10c^3} - \frac{bde^3x^3}{3c} - \frac{be^4x^4}{20c} \\
&\quad + \frac{(d+ex)^5(a+b\arctan(cx))}{5e} - \frac{b\int \frac{c^4d^5-10c^2d^3e^2+5de^4+e(5c^4d^4-10c^2d^2e^2+e^4)x}{1+c^2x^2}\,dx}{5c^3e} \\
&= -\frac{bde(2c^2d^2-e^2)x}{c^3} - \frac{be^2(10c^2d^2-e^2)x^2}{10c^3} - \frac{bde^3x^3}{3c} - \frac{be^4x^4}{20c} + \frac{(d+ex)^5(a+b\arctan(cx))}{5e} \\
&\quad - \frac{(b(5c^4d^4-10c^2d^2e^2+e^4))\int \frac{x}{1+c^2x^2}\,dx}{5c^3} - \frac{(bd(c^4d^4-10c^2d^2e^2+5e^4))\int \frac{1}{1+c^2x^2}\,dx}{5c^3e} \\
&= -\frac{bde(2c^2d^2-e^2)x}{c^3} - \frac{be^2(10c^2d^2-e^2)x^2}{10c^3} - \frac{bde^3x^3}{3c} \\
&\quad - \frac{be^4x^4}{20c} - \frac{bd(c^4d^4-10c^2d^2e^2+5e^4)\arctan(cx)}{5c^4e} \\
&\quad + \frac{(d+ex)^5(a+b\arctan(cx))}{5e} - \frac{b(5c^4d^4-10c^2d^2e^2+e^4)\log(1+c^2x^2)}{10c^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.39

$$\int (d+ex)^4(a+b\arctan(cx))\,dx = \frac{(d+ex)^5(a+b\arctan(cx)) - \frac{b\left(c^2e^2x\left(-6e^2(10d+ex)+c^2\left(120d^3+60d^2ex+20de^2x^2+3e^3x^3\right)\right)+6\left(-10c^2d^2e^2\left(\sqrt{-c^2}d+e\right)+e^4\left(5\sqrt{-}\right.\right.\right.}{}}{5e}$$

```
[In] Integrate[(d + e*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d + e*x)^5*(a + b*ArcTan[c*x]) - (b*(c^2*e^2*x*(-6*e^2*(10*d + e*x) + c^2
*(120*d^3 + 60*d^2*e*x + 20*d*e^2*x^2 + 3*e^3*x^3)) + 6*(-10*c^2*d^2*e^2*(S
qrt[-c^2]*d + e) + e^4*(5*Sqrt[-c^2]*d + e) + c^4*d^4*(Sqrt[-c^2]*d + 5*e))
*Log[1 - Sqrt[-c^2]*x] - 6*(c^4*d^4*(Sqrt[-c^2]*d - 5*e) - 10*c^2*d^2*(Sqrt
[-c^2]*d - e)*e^2 + (5*Sqrt[-c^2]*d - e)*e^4)*Log[1 + Sqrt[-c^2]*x]))/(12*c
^5))/(5*e)
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.34

method	result
parts	$\frac{a(ex+d)^5}{5e}+\frac{b\left(\frac{c\,e^4\arctan(cx)x^5}{5}+c\,e^3\arctan(cx)x^4d+2c\,e^2\arctan(cx)x^3d^2+2ce\arctan(cx)x^2d^3+\arctan(cx)cx\,d^4+\frac{c\arc\ldots}{}\right.}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^5}{5c^4e}+\frac{b\left(\frac{\arctan(cx)c^5d^5}{5e}+\arctan(cx)c^5d^4x+2e\arctan(cx)c^5d^3x^2+2e^2\arctan(cx)c^5d^2x^3+e^3\arctan(cx)c^5d\,x^4+\frac{e^4\arctan(cx)c\ldots}{5}\right.}{}}{}$
default	$\frac{\frac{a(cex+cd)^5}{5c^4e}+\frac{b\left(\frac{\arctan(cx)c^5d^5}{5e}+\arctan(cx)c^5d^4x+2e\arctan(cx)c^5d^3x^2+2e^2\arctan(cx)c^5d^2x^3+e^3\arctan(cx)c^5d\,x^4+\frac{e^4\arctan(cx)c\ldots}{5}\right.}{}}{}$
parallelrisch	$-\frac{-12x^5\arctan(cx)b\,c^5e^4-12x^5a\,c^5e^4-60x^4\arctan(cx)b\,c^5d\,e^3-60x^4a\,c^5d\,e^3-120x^3\arctan(cx)b\,c^5d^2e^2+3x^4b\,c^4e^4-12\ldots}{}$
risch	$\frac{ib\,d^5\ln(c^2x^2+1)}{20e}+ieb\,d^3x^2\ln(-icx+1)-\frac{i(ex+d)^5b\ln(icx+1)}{10e}+\frac{ib\,d^4x\ln(-icx+1)}{2}+\frac{x^5e^4a}{5}+\frac{ie^3bd\,x\ldots}{}$

```
[In] int((e*x+d)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*(e*x+d)^5/e+b/c*(1/5*c*e^4*arctan(c*x)*x^5+c*e^3*arctan(c*x)*x^4*d+2*
c*e^2*arctan(c*x)*x^3*d^2+2*c*e*arctan(c*x)*x^2*d^3+arctan(c*x)*c*x*d^4+1/5
*c/e*arctan(c*x)*d^5-1/5/c^4/e*(10*c^4*d^3*e^2*x+5*c^4*d^2*e^3*x^2+5/3*c^4*
d*e^4*x^3+1/4*e^5*c^4*x^4-5*c^2*d*e^4*x-1/2*e^5*c^2*x^2+1/2*(5*c^4*d^4*e-10
*c^2*d^2*e^3+e^5)*ln(c^2*x^2+1)+(c^5*d^5-10*c^3*d^3*e^2+5*c*d*e^4)*arctan(c
*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43

$$\int (d+ex)^4(a+b\arctan(cx))\,dx$$
$$=\frac{12\,ac^5e^4x^5+3\,(20\,ac^5de^3-bc^4e^4)x^4+20\,(6\,ac^5d^2e^2-bc^4de^3)x^3+6\,(20\,ac^5d^3e-10\,bc^4d^2e^2+bc^2e^4)x^2+6\ldots}{}$$

```
[In] integrate((e*x+d)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*e^4*x^5 + 3*(20*a*c^5*d*e^3 - b*c^4*e^4)*x^4 + 20*(6*a*c^5*d
^2*e^2 - b*c^4*d*e^3)*x^3 + 6*(20*a*c^5*d^3*e - 10*b*c^4*d^2*e^2 + b*c^2*e^
4)*x^2 + 60*(a*c^5*d^4 - 2*b*c^4*d^3*e + b*c^2*d*e^3)*x + 12*(b*c^5*e^4*x^5
 + 5*b*c^5*d*e^3*x^4 + 10*b*c^5*d^2*e^2*x^3 + 10*b*c^5*d^3*e*x^2 + 5*b*c^5*
d^4*x + 10*b*c^3*d^3*e - 5*b*c*d*e^3)*arctan(c*x) - 6*(5*b*c^4*d^4 - 10*b*c
^2*d^2*e^2 + b*e^4)*log(c^2*x^2 + 1))/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(170) = 340$.

Time = 0.45 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.88

$$\int (d+ex)^4(a+b\arctan(cx))\,dx$$
$$= \begin{cases} ad^4x + 2ad^3ex^2 + 2ad^2e^2x^3 + ade^3x^4 + \frac{ae^4x^5}{5} + bd^4x\,\mathrm{atan}\,(cx) + 2bd^3ex^2\,\mathrm{atan}\,(cx) + 2bd^2e^2x^3\,\mathrm{atan}\,(cx) \\ a\left(d^4x + 2d^3ex^2 + 2d^2e^2x^3 + de^3x^4 + \frac{e^4x^5}{5}\right) \end{cases}$$

```
[In] integrate((e*x+d)**4*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 2*a*d**3*e*x**2 + 2*a*d**2*e**2*x**3 + a*d*e**3*x**4
+ a*e**4*x**5/5 + b*d**4*x*atan(c*x) + 2*b*d**3*e*x**2*atan(c*x) + 2*b*d**2
*e**2*x**3*atan(c*x) + b*d*e**3*x**4*atan(c*x) + b*e**4*x**5*atan(c*x)/5 -
b*d**4*log(x**2 + c**(-2))/(2*c) - 2*b*d**3*e*x/c - b*d**2*e**2*x**2/c - b*
d*e**3*x**3/(3*c) - b*e**4*x**4/(20*c) + 2*b*d**3*e*atan(c*x)/c**2 + b*d**2
*e**2*log(x**2 + c**(-2))/c**3 + b*d*e**3*x/c**3 + b*e**4*x**2/(10*c**3) -
b*d*e**3*atan(c*x)/c**4 - b*e**4*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)),
(a*(d**4*x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d*e**3*x**4 + e**4*x**5/5),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.37

$$\int (d+ex)^4(a+b\arctan(cx))\,dx$$
$$=\frac{1}{5}ae^4x^5+ade^3x^4+2\,ad^2e^2x^3+2\,ad^3ex^2+2\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd^3e$$
$$+\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^2e^2$$
$$+\frac{1}{3}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bde^3$$
$$+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)be^4$$
$$+ad^4x+\frac{(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right))bd^4}{2\,c}$$

```
[In] integrate((e*x+d)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^4*x^5 + a*d*e^3*x^4 + 2*a*d^2*e^2*x^3 + 2*a*d^3*e*x^2 + 2*(x^2*arct
an(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^3*e + (2*x^3*arctan(c*x) - c*(x^
2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2*e^2 + 1/3*(3*x^4*arctan(c*x) - c*((c^2
*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d*e^3 + 1/20*(4*x^5*arctan(c*x) - c
*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*e^4 + a*d^4*x + 1/2*(2
*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^4/c
```

Giac [F]

$$\int (d+ex)^4(a+b\arctan(cx))\,dx=\int \left(ex+d\right)^4\left(b\arctan\left(cx\right)+a\right)dx$$

```
[In] integrate((e*x+d)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.48

$$\int (d+ex)^4(a+b\arctan(cx))\,dx = \frac{a\,e^4\,x^5}{5}+a\,d^4\,x-\frac{b\,d^4\,\ln\left(c^2\,x^2+1\right)}{2\,c}-\frac{b\,e^4\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}+2\,a\,d^2\,e^2\,x^3-\frac{b\,e^4\,x^4}{20\,c}+\frac{b\,e^4\,x^2}{10\,c^3}+b\,d^4\,x\,\mathrm{atan}(c\,x)+2\,a\,d^3\,e\,x^2+a\,d\,e^3\,x^4+\frac{b\,e^4\,x^5\,\mathrm{atan}(c\,x)}{5}-\frac{2\,b\,d^3\,e\,x}{c}+\frac{b\,d\,e^3\,x}{c^3}+\frac{2\,b\,d^3\,e\,\mathrm{atan}(c\,x)}{c^2}-\frac{b\,d\,e^3\,\mathrm{atan}(c\,x)}{c^4}+2\,b\,d^3\,e\,x^2\,\mathrm{atan}(c\,x)+b\,d\,e^3\,x^4\,\mathrm{atan}(c\,x)-\frac{b\,d\,e^3\,x^3}{3\,c}+2\,b\,d^2\,e^2\,x^3\,\mathrm{atan}(c\,x)+\frac{b\,d^2\,e^2\,\ln\left(c^2\,x^2+1\right)}{c^3}-\frac{b\,d^2\,e^2\,x^2}{c}$$

```
[In] int((a + b*atan(c*x))*(d + e*x)^4,x)
```

```
[Out] (a*e^4*x^5)/5 + a*d^4*x - (b*d^4*log(c^2*x^2 + 1))/(2*c) - (b*e^4*log(c^2*x
^2 + 1))/(10*c^5) + 2*a*d^2*e^2*x^3 - (b*e^4*x^4)/(20*c) + (b*e^4*x^2)/(10*
c^3) + b*d^4*x*atan(c*x) + 2*a*d^3*e*x^2 + a*d*e^3*x^4 + (b*e^4*x^5*atan(c*
x))/5 - (2*b*d^3*e*x)/c + (b*d*e^3*x)/c^3 + (2*b*d^3*e*atan(c*x))/c^2 - (b*
d*e^3*atan(c*x))/c^4 + 2*b*d^3*e*x^2*atan(c*x) + b*d*e^3*x^4*atan(c*x) - (b
*d*e^3*x^3)/(3*c) + 2*b*d^2*e^2*x^3*atan(c*x) + (b*d^2*e^2*log(c^2*x^2 + 1)
)/c^3 - (b*d^2*e^2*x^2)/c
```

3.2 $\int (d+ex)^3(a+b\arctan(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 144

$$\int (d+ex)^3(a+b\arctan(cx))\,dx = -\frac{be(6c^2d^2-e^2)\,x}{4c^3}-\frac{bde^2x^2}{2c}-\frac{be^3x^3}{12c}-\frac{b(c^4d^4-6c^2d^2e^2+e^4)\arctan(cx)}{4c^4e}+\frac{(d+ex)^4(a+b\arctan(cx))}{4e}-\frac{bd(cd-e)(cd+e)\log(1+c^2x^2)}{2c^3}$$

```
[Out] -1/4*b*e*(6*c^2*d^2-e^2)*x/c^3-1/2*b*d*e^2*x^2/c-1/12*b*e^3*x^3/c-1/4*b*(c^
4*d^4-6*c^2*d^2*e^2+e^4)*arctan(c*x)/c^4/e+1/4*(e*x+d)^4*(a+b*arctan(c*x))/
e-1/2*b*d*(c*d-e)*(c*d+e)*ln(c^2*x^2+1)/c^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4972, 716, 649, 209, 266}

$$\int (d+ex)^3(a+b\arctan(cx))\,dx = \frac{(d+ex)^4(a+b\arctan(cx))}{4e}-\frac{b\arctan(cx)\left(c^4d^4-6c^2d^2e^2+e^4\right)}{4c^4e}-\frac{bex(6c^2d^2-e^2)}{4c^3}-\frac{bd(cd-e)(cd+e)\log(c^2x^2+1)}{2c^3}-\frac{bde^2x^2}{2c}-\frac{be^3x^3}{12c}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/4*(b*e*(6*c^2*d^2 - e^2)*x)/c^3 - (b*d*e^2*x^2)/(2*c) - (b*e^3*x^3)/(12*
c) - (b*(c^4*d^4 - 6*c^2*d^2*e^2 + e^4)*ArcTan[c*x])/(4*c^4*e) + ((d + e*x)
^4*(a + b*ArcTan[c*x]))/(4*e) - (b*d*(c*d - e)*(c*d + e)*Log[1 + c^2*x^2])/
(2*c^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d+ex)^4(a+b\arctan(cx))}{4e}-\frac{(bc)\int\frac{(d+ex)^4}{1+c^2x^2}\,dx}{4e}\\ &= \frac{(d+ex)^4(a+b\arctan(cx))}{4e}\\ &\quad-\frac{(bc)\int\left(\frac{e^2(6c^2d^2-e^2)}{c^4}+\frac{4de^3x}{c^2}+\frac{e^4x^2}{c^2}+\frac{c^4d^4-6c^2d^2e^2+e^4+4c^2d(cd-e)e(cd+e)x}{c^4(1+c^2x^2)}\right)dx}{4e}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{be(6c^2d^2-e^2)\,x}{4c^3} - \frac{bde^2x^2}{2c} - \frac{be^3x^3}{12c} + \frac{(d+ex)^4(a+b\arctan(cx))}{4e} \\
&\quad - \frac{b\int \frac{c^4d^4-6c^2d^2e^2+e^4+4c^2d(cd-e)e(cd+e)x}{1+c^2x^2}\,dx}{4c^3e} \\
&= -\frac{be(6c^2d^2-e^2)\,x}{4c^3} - \frac{bde^2x^2}{2c} - \frac{be^3x^3}{12c} + \frac{(d+ex)^4(a+b\arctan(cx))}{4e} \\
&\quad - \frac{(bd(cd-e)(cd+e))\int \frac{x}{1+c^2x^2}\,dx}{c} - \frac{(b(c^4d^4-6c^2d^2e^2+e^4))\int \frac{1}{1+c^2x^2}\,dx}{4c^3e} \\
&= -\frac{be(6c^2d^2-e^2)\,x}{4c^3} - \frac{bde^2x^2}{2c} - \frac{be^3x^3}{12c} - \frac{b(c^4d^4-6c^2d^2e^2+e^4)\arctan(cx)}{4c^4e} \\
&\quad + \frac{(d+ex)^4(a+b\arctan(cx))}{4e} - \frac{bd(cd-e)(cd+e)\log(1+c^2x^2)}{2c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.51

$$
\int (d+ex)^3(a+b\arctan(cx))\,dx
$$

$$
= \frac{(d+ex)^4(a+b\arctan(cx)) - \frac{bc\left(2\sqrt{-c^2}e^2x\left(-3e^2+c^2\left(18d^2+6dex+e^2x^2\right)\right)-3\left(c^4d^4+e^3\left(4\sqrt{-c^2}d+e\right)-2c^2d^2e\left(2\sqrt{-c^2}d+3e\right)\right)\log\left(1\right.}{6(-c^2)^{5/2}}}{4e}
$$

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d + e*x)^4*(a + b*ArcTan[c*x]) - (b*c*(2*Sqrt[-c^2]*e^2*x*(-3*e^2 + c^2*(
18*d^2 + 6*d*e*x + e^2*x^2)) - 3*(c^4*d^4 + e^3*(4*Sqrt[-c^2]*d + e) - 2*c^
2*d^2*e*(2*Sqrt[-c^2]*d + 3*e))*Log[1 - Sqrt[-c^2]*x] + 3*(c^4*d^4 + 2*c^2*
d^2*(2*Sqrt[-c^2]*d - 3*e)*e + e^3*(-4*Sqrt[-c^2]*d + e))*Log[1 + Sqrt[-c^2
]*x]))/(6*(-c^2)^(5/2)))/(4*e)
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.33

method	result
parts	$\frac{a(ex+d)^4}{4e}+\frac{b\left(\frac{c\,e^3\arctan(cx)x^4}{4}+c\,e^2\arctan(cx)x^3d+\frac{3ce\arctan(cx)x^2d^2}{2}+\arctan(cx)cx\,d^3+\frac{c\arctan(cx)d^4}{4e}-\frac{6c^3d^2e^2x+}{}\right.}{c}$
derivativedivides	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\arctan(cx)c^4d^4}{4e}+\arctan(cx)c^4d^3x+\frac{3e\arctan(cx)c^4d^2x^2}{2}+e^2\arctan(cx)c^4d\,x^3+\frac{e^3\arctan(cx)c^4x^4}{4}-\frac{6c^3d^2e^2x+2e}{}\right.}{c^3}}{c}$
default	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\arctan(cx)c^4d^4}{4e}+\arctan(cx)c^4d^3x+\frac{3e\arctan(cx)c^4d^2x^2}{2}+e^2\arctan(cx)c^4d\,x^3+\frac{e^3\arctan(cx)c^4x^4}{4}-\frac{6c^3d^2e^2x+2e}{}\right.}{c^3}}{c}$
parallelrisch	$-\frac{-3x^4\arctan(cx)b\,c^4e^3-3x^4a\,c^4e^3-12x^3\arctan(cx)b\,c^4d\,e^2-12x^3a\,c^4d\,e^2-18x^2\arctan(cx)b\,c^4d^2e+x^3b\,c^3e^3-18x^2a}{}$
risch	$\frac{3ieb\,d^2x^2\ln(-icx+1)}{4}+\frac{ie^3b\,x^4\ln(-icx+1)}{8}+\frac{ib\,d^3x\ln(-icx+1)}{2}-\frac{i(ex+d)^4b\ln(icx+1)}{8e}+\frac{ib\,d^4\ln(c^2x^2+1)}{16e}-$

```
[In] int((e*x+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*(e*x+d)^4/e+b/c*(1/4*c*e^3*arctan(c*x)*x^4+c*e^2*arctan(c*x)*x^3*d+3/
2*c*e*arctan(c*x)*x^2*d^2+arctan(c*x)*c*x*d^3+1/4*c/e*arctan(c*x)*d^4-1/4/c
^3/e*(6*c^3*d^2*e^2*x+2*e^3*c^3*d*x^2+1/3*e^4*c^3*x^3-c*e^4*x+1/2*(4*c^3*d^
3*e-4*c*d*e^3)*ln(c^2*x^2+1)+(c^4*d^4-6*c^2*d^2*e^2+e^4)*arctan(c*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int (d+ex)^3(a+b\arctan(cx))\,dx$$
$$=\frac{3\,ac^4e^3x^4+(12\,ac^4de^2-bc^3e^3)x^3+6\,(3\,ac^4d^2e-bc^3de^2)x^2+3\,(4\,ac^4d^3-6\,bc^3d^2e+bce^3)x+3\,(bc^4e^3x^4}{12\,c^4}$$

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*e^3*x^4 + (12*a*c^4*d*e^2 - b*c^3*e^3)*x^3 + 6*(3*a*c^4*d^2*e
 - b*c^3*d*e^2)*x^2 + 3*(4*a*c^4*d^3 - 6*b*c^3*d^2*e + b*c*e^3)*x + 3*(b*c^
4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^4*d^3*x + 6*b*c^2
*d^2*e - b*e^3)*arctan(c*x) - 6*(b*c^3*d^3 - b*c*d*e^2)*log(c^2*x^2 + 1))/c
^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(129) = 258.

Time = 0.37 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.82

$$\int (d+ex)^3(a+b\arctan(cx))\,dx = \begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{atan}(cx) + \frac{3bd^2ex^2\operatorname{atan}(cx)}{2} + bde^2x^3\operatorname{atan}(cx) + \frac{be^3x^4\operatorname{atan}(cx)}{4} - \frac{bd^3 \cdots}{} \\ a\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

```
[In] integrate((e*x+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*atan(c*x) + 3*b*d**2*e*x**2*atan(c*x)/2 + b*d*e**2*x**3*atan(c*x) +
 b*e**3*x**4*atan(c*x)/4 - b*d**3*log(x**2 + c**(-2))/(2*c) - 3*b*d**2*e*x/
(2*c) - b*d*e**2*x**2/(2*c) - b*e**3*x**3/(12*c) + 3*b*d**2*e*atan(c*x)/(2*
c**2) + b*d*e**2*log(x**2 + c**(-2))/(2*c**3) + b*e**3*x/(4*c**3) - b*e**3*
atan(c*x)/(4*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 +
 e**3*x**4/4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.29

$$\begin{aligned}\int (d+ex)^3(a+b\arctan(cx))\,dx &= \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^2e \\ &+ \frac{1}{2}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bde^2 \\ &+ \frac{1}{12}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)be^3 \\ &+ ad^3x + \frac{(2cx\arctan(cx) - \log(c^2x^2+1))bd^3}{2c}\end{aligned}$$

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arctan(c*x) - c*(x
/c^2 - arctan(c*x)/c^3))*b*d^2*e + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - lo
g(c^2*x^2 + 1)/c^4))*b*d*e^2 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)
/c^4 + 3*arctan(c*x)/c^5))*b*e^3 + a*d^3*x + 1/2*(2*c*x*arctan(c*x) - log(c
^2*x^2 + 1))*b*d^3/c
```

Giac [F]

$$\int (d+ex)^3(a+b\arctan(cx))\,dx = \int (ex+d)^3(b\arctan(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.37

$$\int (d+ex)^3(a+b\arctan(cx))\,dx = \frac{a\,e^3\,x^4}{4} + a\,d^3\,x - \frac{b\,d^3\,\ln\left(c^2\,x^2+1\right)}{2\,c} - \frac{b\,e^3\,x^3}{12\,c} + b\,d^3\,x\,\mathrm{atan}(c\,x) + \frac{3\,a\,d^2\,e\,x^2}{2} + a\,d\,e^2\,x^3 + \frac{b\,e^3\,x}{4\,c^3} - \frac{b\,e^3\,\mathrm{atan}(c\,x)}{4\,c^4} + \frac{b\,e^3\,x^4\,\mathrm{atan}(c\,x)}{4} - \frac{3\,b\,d^2\,e\,x}{2\,c} + \frac{3\,b\,d^2\,e\,\mathrm{atan}(c\,x)}{2\,c^2} + \frac{3\,b\,d^2\,e\,x^2\,\mathrm{atan}(c\,x)}{2} + b\,d\,e^2\,x^3\,\mathrm{atan}(c\,x) + \frac{b\,d\,e^2\,\ln\left(c^2\,x^2+1\right)}{2\,c^3} - \frac{b\,d\,e^2\,x^2}{2\,c}$$

```
[In] int((a + b*atan(c*x))*(d + e*x)^3,x)
```

```
[Out] (a*e^3*x^4)/4 + a*d^3*x - (b*d^3*log(c^2*x^2 + 1))/(2*c) - (b*e^3*x^3)/(12*
c) + b*d^3*x*atan(c*x) + (3*a*d^2*e*x^2)/2 + a*d*e^2*x^3 + (b*e^3*x)/(4*c^3
) - (b*e^3*atan(c*x))/(4*c^4) + (b*e^3*x^4*atan(c*x))/4 - (3*b*d^2*e*x)/(2*
c) + (3*b*d^2*e*atan(c*x))/(2*c^2) + (3*b*d^2*e*x^2*atan(c*x))/2 + b*d*e^2*
x^3*atan(c*x) + (b*d*e^2*log(c^2*x^2 + 1))/(2*c^3) - (b*d*e^2*x^2)/(2*c)
```

3.3 $\int (d+ex)^2(a+b\arctan(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int (d+ex)^2(a+b\arctan(cx))\,dx = -\frac{bdex}{c}-\frac{be^2x^2}{6c}-\frac{bd\left(d^2-\frac{3e^2}{c^2}\right)\arctan(cx)}{3e}+\frac{(d+ex)^3(a+b\arctan(cx))}{3e}-\frac{b(3c^2d^2-e^2)\log(1+c^2x^2)}{6c^3}$$

```
[Out] -b*d*e*x/c-1/6*b*e^2*x^2/c-1/3*b*d*(d^2-3*e^2/c^2)*arctan(c*x)/e+1/3*(e*x+d
)^3*(a+b*arctan(c*x))/e-1/6*b*(3*c^2*d^2-e^2)*ln(c^2*x^2+1)/c^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4972, 716, 649, 209, 266}

$$\int (d+ex)^2(a+b\arctan(cx))\,dx = \frac{(d+ex)^3(a+b\arctan(cx))}{3e}-\frac{bd\arctan(cx)\left(d^2-\frac{3e^2}{c^2}\right)}{3e}-\frac{b(3c^2d^2-e^2)\log(c^2x^2+1)}{6c^3}-\frac{bdex}{c}-\frac{be^2x^2}{6c}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*d*e*x)/c) - (b*e^2*x^2)/(6*c) - (b*d*(d^2 - (3*e^2)/c^2)*ArcTan[c*x])/
(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x]))/(3*e) - (b*(3*c^2*d^2 - e^2)*Log[
1 + c^2*x^2])/(6*c^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^3(a+b\arctan(cx))}{3e} - \frac{(bc)\int\frac{(d+ex)^3}{1+c^2x^2}\,dx}{3e} \\
&= \frac{(d+ex)^3(a+b\arctan(cx))}{3e} - \frac{(bc)\int\left(\frac{3de^2}{c^2}+\frac{e^3x}{c^2}+\frac{c^2d^3-3de^2+e(3c^2d^2-e^2)x}{c^2(1+c^2x^2)}\right)dx}{3e} \\
&= -\frac{bdex}{c} - \frac{be^2x^2}{6c} + \frac{(d+ex)^3(a+b\arctan(cx))}{3e} - \frac{b\int\frac{c^2d^3-3de^2+e(3c^2d^2-e^2)x}{1+c^2x^2}\,dx}{3ce} \\
&= -\frac{bdex}{c} - \frac{be^2x^2}{6c} + \frac{(d+ex)^3(a+b\arctan(cx))}{3e} \\
&\quad -\frac{1}{3}\left(bd\left(\frac{cd^2}{e}-\frac{3e}{c}\right)\right)\int\frac{1}{1+c^2x^2}\,dx - \frac{(b(3c^2d^2-e^2))\int\frac{x}{1+c^2x^2}\,dx}{3c}
\end{aligned}$$

$$= -\frac{bdex}{c} - \frac{be^2x^2}{6c} - \frac{bd\left(d^2 - \frac{3e^2}{c^2}\right)\arctan(cx)}{3e} + \frac{(d+ex)^3(a+b\arctan(cx))}{3e} - \frac{b(3c^2d^2-e^2)\log(1+c^2x^2)}{6c^3}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.58

$$\int (d+ex)^2(a+b\arctan(cx))\,dx$$
$$= \frac{(d+ex)^3(a+b\arctan(cx)) - \frac{b\left(c^2e^2x(6d+ex)+\left(-e^2\left(3\sqrt{-c^2}d+e\right)+c^2d^2\left(\sqrt{-c^2}d+3e\right)\right)\log\left(1-\sqrt{-c^2}x\right)-\left(c^2d^2\left(\sqrt{-c^2}d-3e\right)+e^2\left(\right.\right.}{2c^3}}{3e}$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d + e*x)^3*(a + b*ArcTan[c*x]) - (b*(c^2*e^2*x*(6*d + e*x) + (-(e^2*(3*Sq
rt[-c^2]*d + e)) + c^2*d^2*(Sqrt[-c^2]*d + 3*e))*Log[1 - Sqrt[-c^2]*x] - (c
^2*d^2*(Sqrt[-c^2]*d - 3*e) + e^2*(-3*Sqrt[-c^2]*d + e))*Log[1 + Sqrt[-c^2]
*x]))/(2*c^3))/(3*e)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

method	result
parts	$\frac{a(ex+d)^3}{3e} + \frac{b\,e^2\arctan(cx)x^3}{3} + be\arctan(cx)\,x^2d + b\arctan(cx)\,x\,d^2 - \frac{b\,e^2x^2}{6c} - \frac{bdex}{c} - \frac{b\,d^2\ln(c^2}{2c}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e} + b\arctan(cx)d^2cx + bce\arctan(cx)d\,x^2 + \frac{bc\,e^2\arctan(cx)x^3}{3} - bedx - \frac{b\,e^2x^2}{6} - \frac{b\ln\left(c^2x^2+1\right)d^2}{2} + \frac{b\,e^2\ln\left(c^2x^2+1\right)}{6c^2} + b}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e} + b\arctan(cx)d^2cx + bce\arctan(cx)d\,x^2 + \frac{bc\,e^2\arctan(cx)x^3}{3} - bedx - \frac{b\,e^2x^2}{6} - \frac{b\ln\left(c^2x^2+1\right)d^2}{2} + \frac{b\,e^2\ln\left(c^2x^2+1\right)}{6c^2} + b}{c}$
parallelrisch	$-\frac{-2x^3\arctan(cx)b\,c^3e^2-2x^3a\,c^3e^2-6x^2\arctan(cx)b\,c^3de-6x^2a\,c^3de-6x\arctan(cx)b\,c^3d^2+x^2b\,c^2e^2-6a\,c^3d^2x+3\ln(c^2}{6c^3}$
risch	$-\frac{i(ex+d)^3b\ln(icx+1)}{6e} + ade\,x^2 + x\,d^2a - \frac{bdex}{c} + \frac{x^3e^2a}{3} - \frac{b\,e^2x^2}{6c} + \frac{iebd\,x^2\ln(-icx+1)}{2} + \frac{ib\,d^2x\ln(-icx}{2}$

```
[In] int((e*x+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(e*x+d)^3/e+1/3*b*e^2*arctan(c*x)*x^3+b*e*arctan(c*x)*x^2*d+b*arctan(
c*x)*x*d^2-1/6*b*e^2*x^2/c-b*d*e*x/c-1/2/c*b*d^2*ln(c^2*x^2+1)+1/6/c^3*e^2*
b*ln(c^2*x^2+1)+1/c^2*e*b*d*arctan(c*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int (d+ex)^2(a+b\arctan(cx))\,dx$$
$$=\frac{2\,ac^3e^2x^3+(6\,ac^3de-bc^2e^2)x^2+6\,(ac^3d^2-bc^2de)x+2\,(bc^3e^2x^3+3\,bc^3dex^2+3\,bc^3d^2x+3\,bcde)\arctan}{6\,c^3}$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + (6*a*c^3*d*e - b*c^2*e^2)*x^2 + 6*(a*c^3*d^2 - b*c^2
*d*e)*x + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x + 3*b*c*d*e)*a
rctan(c*x) - (3*b*c^2*d^2 - b*e^2)*log(c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.55

$$\int (d+ex)^2(a+b\arctan(cx))\,dx$$
$$=\begin{cases} ad^2x+adex^2+\frac{ae^2x^3}{3}+bd^2x\,\mathrm{atan}\,(cx)+bdex^2\,\mathrm{atan}\,(cx)+\frac{be^2x^3\,\mathrm{atan}\,(cx)}{3}-\frac{bd^2\log\left(x^2+\frac{1}{c^2}\right)}{2c}-\frac{bdex}{c}-\frac{be^2x^2}{6c}+ \\ a\left(d^2x+dex^2+\frac{e^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*atan(c*x) + b*d
*e*x**2*atan(c*x) + b*e**2*x**3*atan(c*x)/3 - b*d**2*log(x**2 + c**(-2))/(2
*c) - b*d*e*x/c - b*e**2*x**2/(6*c) + b*d*e*atan(c*x)/c**2 + b*e**2*log(x**
2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.22

$$\int (d+ex)^2(a+b\arctan(cx))\,dx = \frac{1}{3}ae^2x^3 + adex^2 + \left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bde + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)be^2 + ad^2x + \frac{(2cx\arctan(cx) - \log(c^2x^2+1))bd^2}{2c}$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))
*b*d*e + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*e^2
 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^2/c
```

Giac [F]

$$\int (d+ex)^2(a+b\arctan(cx))\,dx = \int (ex+d)^2(b\arctan(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

$$\int (d+ex)^2(a+b\arctan(cx))\,dx = \frac{a\,e^2\,x^3}{3} + a\,d^2\,x - \frac{b\,d^2\,\ln(c^2\,x^2+1)}{2\,c} + \frac{b\,e^2\,\ln(c^2\,x^2+1)}{6\,c^3} - \frac{b\,e^2\,x^2}{6\,c} + a\,d\,e\,x^2 + b\,d^2\,x\,\mathrm{atan}(c\,x) + \frac{b\,e^2\,x^3\,\mathrm{atan}(c\,x)}{3} - \frac{b\,d\,e\,x}{c} + \frac{b\,d\,e\,\mathrm{atan}(c\,x)}{c^2} + b\,d\,e\,x^2\,\mathrm{atan}(c\,x)$$

```
[In] int((a + b*atan(c*x))*(d + e*x)^2,x)
```

```
[Out] (a*e^2*x^3)/3 + a*d^2*x - (b*d^2*log(c^2*x^2 + 1))/(2*c) + (b*e^2*log(c^2*x
^2 + 1))/(6*c^3) - (b*e^2*x^2)/(6*c) + a*d*e*x^2 + b*d^2*x*atan(c*x) + (b*e
^2*x^3*atan(c*x))/3 - (b*d*e*x)/c + (b*d*e*atan(c*x))/c^2 + b*d*e*x^2*atan(
c*x)
```

3.4 $\int (d+ex)(a+b\arctan(cx))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 76

$$\int (d+ex)(a+b\arctan(cx))\,dx = -\frac{bex}{2c}-\frac{b\left(d^2-\frac{e^2}{c^2}\right)\arctan(cx)}{2e}+\frac{(d+ex)^2(a+b\arctan(cx))}{2e}-\frac{bd\log\left(1+c^2x^2\right)}{2c}$$

```
[Out] -1/2*b*e*x/c-1/2*b*(d^2-e^2/c^2)*arctan(c*x)/e+1/2*(e*x+d)^2*(a+b*arctan(c*
x))/e-1/2*b*d*ln(c^2*x^2+1)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4972, 716, 649, 209, 266}

$$\int (d+ex)(a+b\arctan(cx))\,dx = \frac{(d+ex)^2(a+b\arctan(cx))}{2e}-\frac{b\arctan(cx)\left(d^2-\frac{e^2}{c^2}\right)}{2e}-\frac{bd\log\left(c^2x^2+1\right)}{2c}-\frac{bex}{2c}$$

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*e*x)/c - (b*(d^2 - e^2/c^2)*ArcTan[c*x])/(2*e) + ((d + e*x)^2*(a +
b*ArcTan[c*x]))/(2*e) - (b*d*Log[1 + c^2*x^2])/(2*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^2(a+b\arctan(cx))}{2e} - \frac{(bc)\int \frac{(d+ex)^2}{1+c^2x^2}\,dx}{2e}$$

$$= \frac{(d+ex)^2(a+b\arctan(cx))}{2e} - \frac{(bc)\int \left(\frac{e^2}{c^2} + \frac{c^2d^2-e^2+2c^2dex}{c^2(1+c^2x^2)}\right)dx}{2e}$$

$$= -\frac{bex}{2c} + \frac{(d+ex)^2(a+b\arctan(cx))}{2e} - \frac{b\int \frac{c^2d^2-e^2+2c^2dex}{1+c^2x^2}\,dx}{2ce}$$

$$= -\frac{bex}{2c} + \frac{(d+ex)^2(a+b\arctan(cx))}{2e} - (bcd)\int \frac{x}{1+c^2x^2}\,dx - \frac{(b(cd-e)(cd+e))\int \frac{1}{1+c^2x^2}\,dx}{2ce}$$

$$= -\frac{bex}{2c} - \frac{b\left(d^2-\frac{e^2}{c^2}\right)\arctan(cx)}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))}{2e} - \frac{bd\log(1+c^2x^2)}{2c}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int (d+ex)(a+b\arctan(cx))\,dx = adx-\frac{bex}{2c}+\frac{1}{2}aex^2+\frac{be\arctan(cx)}{2c^2}+bdx\arctan(cx)+\frac{1}{2}bex^2\arctan(cx)-\frac{bd\log\left(1+c^2x^2\right)}{2c}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] a*d*x - (b*e*x)/(2*c) + (a*e*x^2)/2 + (b*e*ArcTan[c*x])/(2*c^2) + b*d*x*Arc
Tan[c*x] + (b*e*x^2*ArcTan[c*x])/2 - (b*d*Log[1 + c^2*x^2])/(2*c)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\arctan(cx)x^2e}{2}+b\arctan\left(cx\right)xd-\frac{bd\ln\left(c^2x^2+1\right)}{2c}-\frac{bex}{2c}+\frac{\arctan(cx)be}{2c^2}$
parallelrisch	$-\frac{-\arctan(cx)b\,c^2e\,x^2-a\,c^2e\,x^2-2bdx\arctan(cx)c^2-2a\,c^2dx+bcd\ln\left(c^2x^2+1\right)+bcex-\arctan(cx)be}{2c^2}$
derivativedivides	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b\left(\arctan(cx)d\,c^2x+\frac{\arctan(cx)e\,c^2x^2}{2}-\frac{cex}{2}-\frac{dc\ln\left(c^2x^2+1\right)}{2}+\frac{e\arctan(cx)}{2}\right)}{c}}{c}$
default	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b\left(\arctan(cx)d\,c^2x+\frac{\arctan(cx)e\,c^2x^2}{2}-\frac{cex}{2}-\frac{dc\ln\left(c^2x^2+1\right)}{2}+\frac{e\arctan(cx)}{2}\right)}{c}}{c}$
risch	$-\frac{ib\left(e\,x^2+2dx\right)\ln(icx+1)}{4}+\frac{ibe\,x^2\ln(-icx+1)}{4}+\frac{ibdx\ln(-icx+1)}{2}+\frac{ae\,x^2}{2}+adx-\frac{bd\ln\left(c^2x^2+1\right)}{2c}-\frac{bex}{2c}$

```
[In] int((e*x+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e*x^2+d*x)+1/2*b*arctan(c*x)*x^2*e+b*arctan(c*x)*x*d-1/2*b*d*ln(c^2*
x^2+1)/c-1/2*b*e*x/c+1/2/c^2*arctan(c*x)*b*e
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int (d+ex)(a+b\arctan(cx))\,dx = \frac{ac^2ex^2 - bcd\log\left(c^2x^2+1\right) + (2\,ac^2d - bce)x + \left(bc^2ex^2 + 2\,bc^2dx + be\right)\arctan\left(cx\right)}{2\,c^2}$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*e*x^2 - b*c*d*log(c^2*x^2 + 1) + (2*a*c^2*d - b*c*e)*x + (b*c^2*
e*x^2 + 2*b*c^2*d*x + b*e)*arctan(c*x))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

$$\int (d+ex)(a+b\arctan(cx))\,dx = \begin{cases} adx + \frac{aex^2}{2} + bdx\,\mathrm{atan}\,(cx) + \frac{bex^2\,\mathrm{atan}\,(cx)}{2} - \frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \frac{bex}{2c} + \frac{be\,\mathrm{atan}\,(cx)}{2c^2} & \text{for } c \neq 0 \\ a\left(dx + \frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*atan(c*x) + b*e*x**2*atan(c*x)/2 - b*
d*log(x**2 + c**(-2))/(2*c) - b*e*x/(2*c) + b*e*atan(c*x)/(2*c**2), Ne(c, 0
)), (a*(d*x + e*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int (d+ex)(a+b\arctan(cx))\,dx = \frac{1}{2}\,aex^2 + \frac{1}{2}\left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)be + adx + \frac{\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bd}{2\,c}$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*e + a*d
*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

Giac [F]

$$\int (d+ex)(a+b\arctan(cx))\,dx = \int (ex+d)(b\arctan(cx)+a)\,dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int (d+ex)(a+b\arctan(cx))\,dx = a\,d\,x + \frac{a\,e\,x^2}{2} + b\,d\,x\,\mathrm{atan}(c\,x) - \frac{b\,e\,x}{2\,c} + \frac{b\,e\,\mathrm{atan}(c\,x)}{2\,c^2} + \frac{b\,e\,x^2\,\mathrm{atan}(c\,x)}{2} - \frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{2\,c}$$

```
[In] int((a + b*atan(c*x))*(d + e*x),x)
```

```
[Out] a*d*x + (a*e*x^2)/2 + b*d*x*atan(c*x) - (b*e*x)/(2*c) + (b*e*atan(c*x))/(2*
c^2) + (b*e*x^2*atan(c*x))/2 - (b*d*log(c^2*x^2 + 1))/(2*c)
```

3.5 $\int \frac{a+b\arctan(cx)}{d+ex}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = -\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}$$

```
[Out] -(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+
I*e)/(1-I*c*x))/e+1/2*I*b*polylog(2,1-2/(1-I*c*x))/e-1/2*I*b*polylog(2,1-2*
c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4966, 2449, 2352, 2497}

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e} - \frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}$$

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + ((I/2)*b*PolyLog[2, 1 - 2/(1
- I*c*x)])/e - ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*
c*x))])/e
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} \\
&\quad+\frac{(bc)\int\frac{\log\left(\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx}{e}-\frac{(bc)\int\frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{1+c^2x^2}\,dx}{e} \\
&= -\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} \\
&\quad-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}+\frac{(ib)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-icx}\right)}{e}
\end{aligned}$$

$$= -\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e} - \frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \frac{2a\log(d+ex)+ib\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)-ib\log(1+icx)\log\left(\frac{c(d+ex)}{cd+ie}\right)+ib\operatorname{PolyLog}\left(2,\frac{e(1-icx)}{icd+e}\right)-ib}{2e}$$

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x),x]
```

```
[Out] (2*a*Log[d + e*x] + I*b*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] - I*b
*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + I*b*PolyLog[2, (e*(1 - I*c
*x))/(I*c*d + e)] - I*b*PolyLog[2, -((e*(-I + c*x))/(c*d + I*e))])/(2*e)
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\left(\frac{c\ln(cex+cd)\arctan(cx)}{e}-c\left(-\frac{i\ln(cex+cd)\left(\ln\left(\frac{-cex+ie}{cd+ie}\right)-\ln\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}-\frac{i\left(\operatorname{dilog}\left(\frac{-cex+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{cex+ie}{-cd+ie}\right.\right.}{2e}\right.\right.}{c}$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\ln(cex+cd)\arctan(cx)}{e}-\frac{i\ln(cex+cd)\left(-\ln\left(\frac{-cex+ie}{cd+ie}\right)+\ln\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}+\frac{i\left(\operatorname{dilog}\left(\frac{-cex+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}\right)}{c}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\ln(cex+cd)\arctan(cx)}{e}-\frac{i\ln(cex+cd)\left(-\ln\left(\frac{-cex+ie}{cd+ie}\right)+\ln\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}+\frac{i\left(\operatorname{dilog}\left(\frac{-cex+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}\right)}{c}$
risch	$\frac{ib\operatorname{dilog}\left(\frac{-idc+e(-icx+1)-e}{-idc-e}\right)}{2e}+\frac{ib\ln(-icx+1)\ln\left(\frac{-idc+e(-icx+1)-e}{-idc-e}\right)}{2e}+\frac{a\ln(idc-e(-icx+1)+e)}{e}-\frac{ib\operatorname{dilog}\left(\frac{idc+e(i}{idc}\right.}{2e}$

```
[In] int((a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(e*x+d)/e+b/c*(c*ln(c*e*x+c*d)/e*arctan(c*x)-c*(-1/2*I*ln(c*e*x+c*d)*(l
n((I*e-c*e*x)/(c*d+I*e))-ln((I*e+c*e*x)/(I*e-c*d)))/e-1/2*I*(dilog((I*e-c*e
*x)/(c*d+I*e))-dilog((I*e+c*e*x)/(I*e-c*d)))/e))
```

Fricas [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \int \frac{b\arctan\left(cx\right)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \int \frac{a+b\,\mathrm{atan}\left(cx\right)}{d+ex}\,dx$$

```
[In] integrate((a+b*atan(c*x))/(e*x+d),x)
[Out] Integral((a + b*atan(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \int \frac{b\arctan\left(cx\right)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
[Out] 2*b*integrate(1/2*arctan(c*x)/(e*x + d), x) + a*log(e*x + d)/e
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \int \frac{b\arctan\left(cx\right)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{d+e\,x}\,dx$$

```
[In] int((a + b*atan(c*x))/(d + e*x),x)
[Out] int((a + b*atan(c*x))/(d + e*x), x)
```

3.6 $\int \frac{a+b\arctan(cx)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{a+b\arctan(cx)}{(d+ex)^2}\,dx=\frac{bc^2d\arctan(cx)}{e\left(c^2d^2+e^2\right)}-\frac{a+b\arctan(cx)}{e(d+ex)}+\frac{bc\log(d+ex)}{c^2d^2+e^2}-\frac{bc\log\left(1+c^2x^2\right)}{2\left(c^2d^2+e^2\right)}$$

```
[Out] b*c^2*d*arctan(c*x)/e/(c^2*d^2+e^2)+(-a-b*arctan(c*x))/e/(e*x+d)+b*c*ln(e*x
+d)/(c^2*d^2+e^2)-1/2*b*c*ln(c^2*x^2+1)/(c^2*d^2+e^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4972, 720, 31, 649, 209, 266}

$$\int \frac{a+b\arctan(cx)}{(d+ex)^2}\,dx=-\frac{a+b\arctan(cx)}{e(d+ex)}+\frac{bc^2d\arctan(cx)}{e\left(c^2d^2+e^2\right)}-\frac{bc\log\left(c^2x^2+1\right)}{2\left(c^2d^2+e^2\right)}+\frac{bc\log(d+ex)}{c^2d^2+e^2}$$

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x)^2,x]
```

```
[Out] (b*c^2*d*ArcTan[c*x])/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])/(e*(d + e*x
)) + (b*c*Log[d + e*x])/(c^2*d^2 + e^2) - (b*c*Log[1 + c^2*x^2])/(2*(c^2*d^
2 + e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx)}{e(d+ex)}+\frac{(bc)\int \frac{1}{(d+ex)(1+c^2x^2)}\,dx}{e}\\
&= -\frac{a+b\arctan(cx)}{e(d+ex)}+\frac{(bc)\int \frac{c^2d-c^2ex}{1+c^2x^2}\,dx}{e\left(c^2d^2+e^2\right)}+\frac{(bce)\int \frac{1}{d+ex}\,dx}{c^2d^2+e^2}\\
&= -\frac{a+b\arctan(cx)}{e(d+ex)}+\frac{bc\log(d+ex)}{c^2d^2+e^2}-\frac{\left(bc^3\right)\int \frac{x}{1+c^2x^2}\,dx}{c^2d^2+e^2}+\frac{\left(bc^3d\right)\int \frac{1}{1+c^2x^2}\,dx}{e\left(c^2d^2+e^2\right)}\\
&= \frac{bc^2d\arctan(cx)}{e\left(c^2d^2+e^2\right)}-\frac{a+b\arctan(cx)}{e(d+ex)}+\frac{bc\log(d+ex)}{c^2d^2+e^2}-\frac{bc\log\left(1+c^2x^2\right)}{2\left(c^2d^2+e^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \frac{a + b\arctan(cx)}{(d + ex)^2}\,dx = \frac{-\frac{a+b\arctan(cx)}{d+ex} + \frac{bc\left(\left(\sqrt{-c^2}d-e\right)\log\left(1-\sqrt{-c^2}x\right)-\left(\sqrt{-c^2}d+e\right)\log\left(1+\sqrt{-c^2}x\right)+2e\log(d+ex)\right)}{2(c^2d^2+e^2)}}{e}$$

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x)^2,x]
```

```
[Out] (-((a + b*ArcTan[c*x])/(d + e*x)) + (b*c*((Sqrt[-c^2]*d - e)*Log[1 - Sqrt[-
c^2]*x] - (Sqrt[-c^2]*d + e)*Log[1 + Sqrt[-c^2]*x] + 2*e*Log[d + e*x]))/(2*
(c^2*d^2 + e^2)))/e
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^2\arctan(cx)}{(cex+cd)e}+\frac{c^2\left(\frac{e\ln(cex+cd)}{c^2d^2+e^2}+\frac{-\frac{e\ln\left(c^2x^2+1\right)}{2}+dc\arctan(cx)}{c^2d^2+e^2}\right)}{e}\right)}{c}$
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\arctan(cx)}{(cex+cd)e}+\frac{\frac{e\ln(cex+cd)}{c^2d^2+e^2}+\frac{-\frac{e\ln\left(c^2x^2+1\right)}{2}+dc\arctan(cx)}{c^2d^2+e^2}}{e}\right)}{c}$
default	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\arctan(cx)}{(cex+cd)e}+\frac{\frac{e\ln(cex+cd)}{c^2d^2+e^2}+\frac{-\frac{e\ln\left(c^2x^2+1\right)}{2}+dc\arctan(cx)}{c^2d^2+e^2}}{e}\right)}{c}$
parallelrisch	$-\frac{-2bdx\arctan(cx)c^4e+\ln\left(c^2x^2+1\right)xb\,c^3e^2-2\ln(ex+d)xb\,c^3e^2+\ln\left(c^2x^2+1\right)b\,c^3de-2\ln(ex+d)b\,c^3de+2a\,c^4d^2+2b\,e^2\,a}{2(ex+d)c^2e(c^2d^2+e^2)}$
risch	$\frac{ib\ln(icx+1)}{2e(ex+d)}+\frac{-ib\,c^2d^2\ln(-icx+1)-ib\,e^2\ln(-icx+1)+2\ln(-ex-d)bc\,e^2x+2\ln(-ex-d)bcde-2a\,c^2d^2-2a\,e^2-\ln\left(\left(-\right.\right.}{}$

```
[In] int((a+b*arctan(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arctan(c*x)+c^2/e*(e/(c^2*d^2+e^2)*ln(
c*e*x+c*d)+1/(c^2*d^2+e^2)*(-1/2*e*ln(c^2*x^2+1)+d*c*arctan(c*x))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

$$\int \frac{a+b\arctan(cx)}{(d+ex)^2}\,dx = -\frac{2\,ac^2d^2+2\,ae^2-2\,(bc^2dex-be^2)\arctan(cx)+(bce^2x+bcde)\log(c^2x^2+1)-2\,(bce^2x+bcde)\log(ex+ \cdots}{2\,(c^2d^3e+de^3+(c^2d^2e^2+e^4)x)}$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*c^2*d^2 + 2*a*e^2 - 2*(b*c^2*d*e*x - b*e^2)*arctan(c*x) + (b*c*e^
2*x + b*c*d*e)*log(c^2*x^2 + 1) - 2*(b*c*e^2*x + b*c*d*e)*log(e*x + d))/(c^
2*d^3*e + d*e^3 + (c^2*d^2*e^2 + e^4)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.71

$$\int \frac{a+b\arctan(cx)}{(d+ex)^2}\,dx = \begin{cases} \frac{ax}{d^2} \\ \frac{ax+bx\operatorname{atan}(cx)-\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{2c}}{d^2} \\ -\frac{a}{de+e^2x} \\ -\frac{2ad}{2d^2e+2de^2x}+\frac{ibd\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^2e+2de^2x}+\frac{ibd}{2d^2e+2de^2x}-\frac{ibex\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^2e+2de^2x} \\ -\frac{2ad}{2d^2e+2de^2x}-\frac{ibd\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^2e+2de^2x}-\frac{ibd}{2d^2e+2de^2x}+\frac{ibex\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^2e+2de^2x} \\ -\frac{2ac^2d^2}{2c^2d^3e+2c^2d^2e^2x+2de^3+2e^4x}-\frac{2ae^2}{2c^2d^3e+2c^2d^2e^2x+2de^3+2e^4x}+\frac{2bc^2dex\operatorname{atan}(cx)}{2c^2d^3e+2c^2d^2e^2x+2de^3+2e^4x}-\frac{bcde\log\left(x^2+\frac{1}{c^2}\right)}{2c^2d^3e+2c^2d^2e^2x+2de^3+2e^4x}+ \cdots \end{cases}$$

```
[In] integrate((a+b*atan(c*x))/(e*x+d)**2,x)
```

```
[Out] Piecewise((a*x/d**2, Eq(c, 0) & Eq(e, 0)), ((a*x + b*x*atan(c*x) - b*log(x*
*2 + c**(-2))/(2*c))/d**2, Eq(e, 0)), (-a/(d*e + e**2*x), Eq(c, 0)), (-2*a*
d/(2*d**2*e + 2*d*e**2*x) + I*b*d*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x) + I*
b*d/(2*d**2*e + 2*d*e**2*x) - I*b*e*x*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x),
 Eq(c, -I*e/d)), (-2*a*d/(2*d**2*e + 2*d*e**2*x) - I*b*d*atanh(e*x/d)/(2*d*
*2*e + 2*d*e**2*x) - I*b*d/(2*d**2*e + 2*d*e**2*x) + I*b*e*x*atanh(e*x/d)/(
2*d**2*e + 2*d*e**2*x), Eq(c, I*e/d)), (-2*a*c**2*d**2/(2*c**2*d**3*e + 2*c
```

```
**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x) - 2*a*e**2/(2*c**2*d**3*e + 2*c**2*d
**2*e**2*x + 2*d*e**3 + 2*e**4*x) + 2*b*c**2*d*e*x*atan(c*x)/(2*c**2*d**3*e
 + 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x) - b*c*d*e*log(x**2 + c**(-2))/
(2*c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x) + 2*b*c*d*e*log(
d/e + x)/(2*c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x) - b*c*e
**2*x*log(x**2 + c**(-2))/(2*c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d*e**3 +
2*e**4*x) + 2*b*c*e**2*x*log(d/e + x)/(2*c**2*d**3*e + 2*c**2*d**2*e**2*x +
 2*d*e**3 + 2*e**4*x) - 2*b*e**2*atan(c*x)/(2*c**2*d**3*e + 2*c**2*d**2*e**
2*x + 2*d*e**3 + 2*e**4*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{(d+ex)^2}\,dx = \frac{1}{2}\left(\left(\frac{2\,cd\arctan(cx)}{c^2d^2e+e^3}-\frac{\log(c^2x^2+1)}{c^2d^2+e^2}+\frac{2\,\log(ex+d)}{c^2d^2+e^2}\right)c-\frac{2\,\arctan(cx)}{e^2x+de}\right)b - \frac{a}{e^2x+de}$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((2*c*d*arctan(c*x)/(c^2*d^2*e + e^3) - log(c^2*x^2 + 1)/(c^2*d^2 + e^2
) + 2*log(e*x + d)/(c^2*d^2 + e^2))*c - 2*arctan(c*x)/(e^2*x + d*e))*b - a/
(e^2*x + d*e)
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex)^2}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \frac{a+b\arctan(cx)}{(d+ex)^2}\,dx$$
$$= \frac{d^2\left(b\,c\,\ln\left(d+e\,x\right)-\frac{b\,c\,\ln\left(c^2\,x^2+1\right)}{2}+a\,c^2\,x+b\,c^2\,x\,\mathrm{atan}(c\,x)\right)-d\,e\left(b\,\mathrm{atan}(c\,x)-b\,c\,x\,\ln\left(d+e\,x\right)+\frac{b\,c\,x\,\ln(}{}\right.}{d\,\left(c^2\,d^2+e^2\right)\,\left(d+e\,x\right)}$$

```
[In] int((a + b*atan(c*x))/(d + e*x)^2,x)

[Out] (d^2*(b*c*log(d + e*x) - (b*c*log(c^2*x^2 + 1))/2 + a*c^2*x + b*c^2*x*atan(
c*x)) - d*e*(b*atan(c*x) - b*c*x*log(d + e*x) + (b*c*x*log(c^2*x^2 + 1))/2)
 + a*e^2*x)/(d*(e^2 + c^2*d^2)*(d + e*x))
```

3.7 $\int \frac{a+b\arctan(cx)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 146

$$\int \frac{a+b\arctan(cx)}{(d+ex)^3}\,dx = -\frac{bc}{2(c^2d^2+e^2)(d+ex)} + \frac{bc^2(cd-e)(cd+e)\arctan(cx)}{2e(c^2d^2+e^2)^2} - \frac{a+b\arctan(cx)}{2e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{(c^2d^2+e^2)^2} - \frac{bc^3d\log(1+c^2x^2)}{2(c^2d^2+e^2)^2}$$

```
[Out] -1/2*b*c/(c^2*d^2+e^2)/(e*x+d)+1/2*b*c^2*(c*d-e)*(c*d+e)*arctan(c*x)/e/(c^2
*d^2+e^2)^2+1/2*(-a-b*arctan(c*x))/e/(e*x+d)^2+b*c^3*d*ln(e*x+d)/(c^2*d^2+e
^2)^2-1/2*b*c^3*d*ln(c^2*x^2+1)/(c^2*d^2+e^2)^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4972, 724, 815, 649, 209, 266}

$$\int \frac{a+b\arctan(cx)}{(d+ex)^3}\,dx = -\frac{a+b\arctan(cx)}{2e(d+ex)^2} + \frac{bc^2\arctan(cx)(cd-e)(cd+e)}{2e(c^2d^2+e^2)^2} - \frac{bc}{2(c^2d^2+e^2)(d+ex)} - \frac{bc^3d\log(c^2x^2+1)}{2(c^2d^2+e^2)^2} + \frac{bc^3d\log(d+ex)}{(c^2d^2+e^2)^2}$$

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*c)/((c^2*d^2 + e^2)*(d + e*x)) + (b*c^2*(c*d - e)*(c*d + e)*ArcTan[
c*x])/(2*e*(c^2*d^2 + e^2)^2) - (a + b*ArcTan[c*x])/(2*e*(d + e*x)^2) + (b*
c^3*d*Log[d + e*x])/(c^2*d^2 + e^2)^2 - (b*c^3*d*Log[1 + c^2*x^2])/(2*(c^2*
d^2 + e^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\arctan(cx)}{2e(d+ex)^2}+\frac{(bc)\int\frac{1}{(d+ex)^2(1+c^2x^2)}\,dx}{2e}$$

$$=-\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\arctan(cx)}{2e(d+ex)^2}+\frac{(bc^3)\int\frac{d-ex}{(d+ex)(1+c^2x^2)}\,dx}{2e\left(c^2d^2+e^2\right)}$$

$$=-\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\arctan(cx)}{2e(d+ex)^2}+\frac{(bc^3)\int\left(\frac{2de^2}{(c^2d^2+e^2)(d+ex)}+\frac{c^2d^2-e^2-2c^2dex}{(c^2d^2+e^2)(1+c^2x^2)}\right)dx}{2e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\arctan(cx)}{2e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{\left(c^2d^2+e^2\right)^2} + \frac{\left(bc^3\right)\int \frac{c^2d^2-e^2-2c^2dex}{1+c^2x^2}\,dx}{2e\left(c^2d^2+e^2\right)^2}$$

$$= -\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\arctan(cx)}{2e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{\left(c^2d^2+e^2\right)^2} - \frac{\left(bc^5d\right)\int \frac{x}{1+c^2x^2}\,dx}{\left(c^2d^2+e^2\right)^2} + \frac{\left(bc^3(cd-e)(cd+e)\right)\int \frac{1}{1+c^2x^2}\,dx}{2e\left(c^2d^2+e^2\right)^2}$$

$$= -\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)} + \frac{bc^2(cd-e)(cd+e)\arctan(cx)}{2e\left(c^2d^2+e^2\right)^2} - \frac{a+b\arctan(cx)}{2e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{\left(c^2d^2+e^2\right)^2} - \frac{bc^3d\log\left(1+c^2x^2\right)}{2\left(c^2d^2+e^2\right)^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.32

$$\int \frac{a+b\arctan(cx)}{(d+ex)^3}\,dx = -\frac{2(a+b\arctan(cx)) + \frac{bc(d+ex)\left(2e\left(c^2d^2+e^2\right)-\left(c^2d\left(\sqrt{-c^2}d-2e\right)-\sqrt{-c^2}e^2\right)(d+ex)\log\left(1-\sqrt{-c^2}x\right)-\left(\sqrt{-c^2}e^2-c^2d\left(\sqrt{-c^2}d+2e\right)\right.\cdots}{\left(c^2d^2+e^2\right)^2}}{4e(d+ex)^2}$$

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/4*(2*(a + b*ArcTan[c*x]) + (b*c*(d + e*x)*(2*e*(c^2*d^2 + e^2) - (c^2*d*
(Sqrt[-c^2]*d - 2*e) - Sqrt[-c^2]*e^2)*(d + e*x)*Log[1 - Sqrt[-c^2]*x] - (S
qrt[-c^2]*e^2 - c^2*d*(Sqrt[-c^2]*d + 2*e))*(d + e*x)*Log[1 + Sqrt[-c^2]*x]
 - 4*c^2*d*e*(d + e*x)*Log[d + e*x]))/(c^2*d^2 + e^2)^2)/(e*(d + e*x)^2)
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05

method	result
parts	$-\frac{a}{2(ex+d)^2 e}+\frac{b\left(-\frac{c^3 \arctan(cx)}{2(cex+cd)^2 e}+\frac{c^3\left(-\frac{e}{(c^2d^2+e^2)(cex+cd)}+\frac{2ecd\ln(cex+cd)}{(c^2d^2+e^2)^2}+\frac{-cde\ln(c^2x^2+1)+(c^2d^2-e^2)\arctan(cx)}{(c^2d^2+e^2)^2}\right)}{2e}\right)}{c}$
derivativedivides	$\frac{-\frac{a c^3}{2(cex+cd)^2 e}+b c^3\left(-\frac{\arctan(cx)}{2(cex+cd)^2 e}+\frac{-\frac{e}{(c^2d^2+e^2)(cex+cd)}+\frac{2ecd\ln(cex+cd)}{(c^2d^2+e^2)^2}+\frac{-cde\ln(c^2x^2+1)+(c^2d^2-e^2)\arctan(cx)}{(c^2d^2+e^2)^2}}{2e}\right)}{c}$
default	$\frac{-\frac{a c^3}{2(cex+cd)^2 e}+b c^3\left(-\frac{\arctan(cx)}{2(cex+cd)^2 e}+\frac{-\frac{e}{(c^2d^2+e^2)(cex+cd)}+\frac{2ecd\ln(cex+cd)}{(c^2d^2+e^2)^2}+\frac{-cde\ln(c^2x^2+1)+(c^2d^2-e^2)\arctan(cx)}{(c^2d^2+e^2)^2}}{2e}\right)}{c}$
parallelrisch	$-\frac{bcde^2+e^3a-x^2ac^4d^2e-2bd^3\arctan(cx)xc^4-x^2bc^3de^2-xbc^3d^2e+3\arctan(cx)bc^2d^2e-2xac^4d^3+\ln(c^2x^2+1)bc^3d^3 \cdots}{\cdots}$
risch	Expression too large to display

```
[In] int((a+b*arctan(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arctan(c*x)+1/2*c^3/e*(-e/
(c^2*d^2+e^2)/(c*e*x+c*d)+2*e*c*d/(c^2*d^2+e^2)^2*ln(c*e*x+c*d)+1/(c^2*d^2+
e^2)^2*(-c*d*e*ln(c^2*x^2+1)+(c^2*d^2-e^2)*arctan(c*x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(138) = 276.

Time = 0.35 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.14

$$\int \frac{a+b\arctan(cx)}{(d+ex)^3}\,dx = -\frac{ac^4d^4+bc^3d^3e+2\,ac^2d^2e^2+bcde^3+ae^4+(bc^3d^2e^2+bce^4)x+(3\,bc^2d^2e^2+be^4-(bc^4d^2e^2-bc^2e^4)x^2 \cdots}{2\,(c^4d^6e+2\,c^2d^4e^3+d^2e^5+\cdots}$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^4*d^4 + b*c^3*d^3*e + 2*a*c^2*d^2*e^2 + b*c*d*e^3 + a*e^4 + (b*c^
3*d^2*e^2 + b*c*e^4)*x + (3*b*c^2*d^2*e^2 + b*e^4 - (b*c^4*d^2*e^2 - b*c^2*
e^4)*x^2 - 2*(b*c^4*d^3*e - b*c^2*d*e^3)*x)*arctan(c*x) + (b*c^3*d*e^3*x^2
+ 2*b*c^3*d^2*e^2*x + b*c^3*d^3*e)*log(c^2*x^2 + 1) - 2*(b*c^3*d*e^3*x^2 +
2*b*c^3*d^2*e^2*x + b*c^3*d^3*e)*log(e*x + d))/(c^4*d^6*e + 2*c^2*d^4*e^3 +
```

```
 d^2*e^5 + (c^4*d^4*e^3 + 2*c^2*d^2*e^5 + e^7)*x^2 + 2*(c^4*d^5*e^2 + 2*c^2
*d^3*e^4 + d*e^6)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.27 (sec) , antiderivative size = 2866, normalized size of antiderivative = 19.63

$$\int \frac{a+b\arctan(cx)}{(d+ex)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*atan(c*x))/(e*x+d)**3,x)
```

```
[Out] Piecewise((a*x/d**3, Eq(c, 0) & Eq(e, 0)), ((a*x + b*x*atan(c*x) - b*log(x*
*2 + c**(-2))/(2*c))/d**3, Eq(e, 0)), (-a/(2*d**2*e + 4*d*e**2*x + 2*e**3*x
**2), Eq(c, 0)), (-4*a*d**2/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2)
+ 3*I*b*d**2*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) +
2*I*b*d**2/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - 2*I*b*d*e*x*ata
nh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) + I*b*d*e*x/(8*d**
4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - I*b*e**2*x**2*atanh(e*x/d)/(8*d*
*4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2), Eq(c, -I*e/d)), (-4*a*d**2/(8*d*
*4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - 3*I*b*d**2*atanh(e*x/d)/(8*d**4
*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - 2*I*b*d**2/(8*d**4*e + 16*d**3*e*
*2*x + 8*d**2*e**3*x**2) + 2*I*b*d*e*x*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**
2*x + 8*d**2*e**3*x**2) - I*b*d*e*x/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**
3*x**2) + I*b*e**2*x**2*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e*
*3*x**2), Eq(c, I*e/d)), (-a*c**4*d**4/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x
+ 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d*
*2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 2*a*c**2*d**2*e**2
/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*
e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*
x + 2*e**7*x**2) - a*e**4/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4
*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2
+ 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*b*c**4*d**3*e*x*atan(c*x)/(2*
c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*e**3
 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x +
2*e**7*x**2) + b*c**4*d**2*e**2*x**2*atan(c*x)/(2*c**4*d**6*e + 4*c**4*d**5
*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4
*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*c**3*d**
3*e*log(x**2 + c**(-2))/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e
**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 +
2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*b*c**3*d**3*e*log(d/e + x)/(2*c
**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*e**3
+ 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2
*e**7*x**2) - b*c**3*d**3*e/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d*
```

```
*4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**
2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 2*b*c**3*d**2*e**2*x*log(x**2
 + c**(-2))/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4
*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5
+ 4*d*e**6*x + 2*e**7*x**2) + 4*b*c**3*d**2*e**2*x*log(d/e + x)/(2*c**4*d**
6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**
2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x
**2) - b*c**3*d**2*e**2*x/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4
*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2
+ 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*c**3*d*e**3*x**2*log(x**2 + c
**(-2))/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**
2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*
d*e**6*x + 2*e**7*x**2) + 2*b*c**3*d*e**3*x**2*log(d/e + x)/(2*c**4*d**6*e
+ 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d*
*3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 - 3*b*c**2*d**2*e**2*atan(c*x)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**
4*d**4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5
*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 2*b*c**2*d*e**3*x*atan(c*
x)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**
4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**
6*x + 2*e**7*x**2) - b*c**2*e**4*x**2*atan(c*x)/(2*c**4*d**6*e + 4*c**4*d**
5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x +
4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*c*d*e**
3/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4
*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6
*x + 2*e**7*x**2) - b*c*e**4*x/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4
*d**4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c**2*d**3*e**4*x + 4*c**2*d**2*e**5*
x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*e**4*atan(c*x)/(2*c**4*d
**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 + 4*c**2*d**4*e**3 + 8*c
**2*d**3*e**4*x + 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7
*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.47

$$\int \frac{a + b\arctan(cx)}{(d + ex)^3}\, dx =$$

$$-\frac{1}{2}\left(\left(\frac{c^2 d\log\left(c^2x^2 + 1\right)}{c^4d^4 + 2\,c^2d^2e^2 + e^4} - \frac{2\,c^2 d\log\left(ex + d\right)}{c^4d^4 + 2\,c^2d^2e^2 + e^4} - \frac{\left(c^4d^2 - c^2e^2\right)\arctan\left(cx\right)}{\left(c^4d^4e + 2\,c^2d^2e^3 + e^5\right)c} + \frac{1}{c^2d^3 + de^2 + (c^2d^2e + e^3\right.\right.$$

$$-\frac{a}{2\left(e^3x^2 + 2\,de^2x + d^2e\right)}$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c^2*d*log(c^2*x^2 + 1)/(c^4*d^4 + 2*c^2*d^2*e^2 + e^4) - 2*c^2*d*log
(e*x + d)/(c^4*d^4 + 2*c^2*d^2*e^2 + e^4) - (c^4*d^2 - c^2*e^2)*arctan(c*x)
/((c^4*d^4*e + 2*c^2*d^2*e^3 + e^5)*c) + 1/(c^2*d^3 + d*e^2 + (c^2*d^2*e +
e^3)*x))*c + arctan(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*b - 1/2*a/(e^3*x^2
+ 2*d*e^2*x + d^2*e)
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex)^3}\,dx=\int \frac{b\arctan(cx)+a}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 591, normalized size of antiderivative = 4.05

$$\int \frac{a+b\arctan(cx)}{(d+ex)^3}\,dx = \frac{\frac{x\left(a\,c^2\,d^2+\frac{b\,c\,d\,e}{2}+a\,e^2\right)}{d\,(c^2\,d^2+e^2)}-\frac{b\,\mathrm{atan}(c\,x)}{2\,e}+\frac{x^2\left(\frac{a\,c^2\,d^2\,e}{2}+\frac{b\,c\,d\,e^2}{2}+\frac{a\,e^3}{2}\right)}{d^2\,(c^2\,d^2+e^2)}+\frac{x^4\left(\frac{a\,c^4\,d^2\,e}{2}+\frac{b\,c^3\,d\,e^2}{2}+\frac{a\,c^2\,e^3}{2}\right)}{d^2\,(c^2\,d^2+e^2)}+\frac{x^3\left(a\,c^4\,d^2+\frac{b\,c^3\,d\,e}{2}+a\,c^2\,e^2\right)}{d\,(c^2\,d^2+e^2)}}{c^2\,d^2\,x^2+2\,c^2\,d\,e\,x^3+c^2\,e^2\,x^4+d^2+2\,d\,e\,x+e^2\,x^2} + \frac{b\,c^3\,d\,\ln(d+e\,x)}{c^4\,d^4+2\,c^2\,d^2\,e^2+e^4}-\frac{b\,c^3\,d\,\ln(c^2\,x^2+1)}{2\,(c^4\,d^4+2\,c^2\,d^2\,e^2+e^4)} + \frac{\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,(c^2)^{7/2}\,(c^4\,d^4+8\,c^2\,d^2\,e^2+2\,e^4)\,(3\,c^6\,d^4+26\,c^4\,d^2\,e^2+4\,c^2\,e^4)\,(27\,b\,c^{10}\,d^{10}+}{2\,c\,(81\,c^{26}\,d^{20}\,e+1662\,c^{24}\,d^{18}\,e^3+11515\,c^{22}\,d^{16}\,e^5+32306\,c^{20}\,d^{14}\,e^7+43705\,c^{18}\,d^{12}\,e^9+28142\,c^{16}\,d^{10}\,e^{1}}$$

```
[In] int((a + b*atan(c*x))/(d + e*x)^3,x)
[Out] ((x*(a*e^2 + a*c^2*d^2 + (b*c*d*e)/2))/(d*(e^2 + c^2*d^2)) - (b*atan(c*x))/
(2*e) + (x^2*((a*e^3)/2 + (b*c*d*e^2)/2 + (a*c^2*d^2*e)/2))/(d^2*(e^2 + c^2
*d^2)) + (x^4*((a*c^2*e^3)/2 + (a*c^4*d^2*e)/2 + (b*c^3*d*e^2)/2))/(d^2*(e^
2 + c^2*d^2)) + (x^3*(a*c^4*d^2 + a*c^2*e^2 + (b*c^3*d*e)/2))/(d*(e^2 + c^2
*d^2)) - (b*c^2*x^2*atan(c*x))/(2*e))/(d^2 + e^2*x^2 + 2*d*e*x + c^2*d^2*x^
2 + c^2*e^2*x^4 + 2*c^2*d*e*x^3) + (b*c^3*d*log(d + e*x))/(e^4 + c^4*d^4 +
2*c^2*d^2*e^2) - (b*c^3*d*log(c^2*x^2 + 1))/(2*(e^4 + c^4*d^4 + 2*c^2*d^2*e
^2)) + (atan((c^2*x)/(c^2)^(1/2))*(c^2)^(7/2)*(2*e^4 + c^4*d^4 + 8*c^2*d^2*
e^2)*(3*c^6*d^4 + 4*c^2*e^4 + 26*c^4*d^2*e^2)*(3*b*e^10 + 27*b*c^10*d^10 +
7*b*c^2*d^2*e^8 - 26*b*c^4*d^4*e^6 - 34*b*c^6*d^6*e^4 + 23*b*c^8*d^8*e^2))/
(2*c*(81*c^26*d^20*e - 24*c^6*e^21 - 380*c^8*d^2*e^19 - 2054*c^10*d^4*e^17
- 3650*c^12*d^6*e^15 + 4857*c^14*d^8*e^13 + 28142*c^16*d^10*e^11 + 43705*c^
18*d^12*e^9 + 32306*c^20*d^14*e^7 + 11515*c^22*d^16*e^5 + 1662*c^24*d^18*e^
3))
```

3.8 $\int \frac{a+b \arctan(cx)}{(d+ex)^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 206

$$\int \frac{a+b\arctan(cx)}{(d+ex)^4}\,dx = -\frac{bc}{6(c^2d^2+e^2)(d+ex)^2} - \frac{2bc^3d}{3(c^2d^2+e^2)^2(d+ex)} + \frac{bc^4d(c^2d^2-3e^2)\arctan(cx)}{3e(c^2d^2+e^2)^3} - \frac{a+b\arctan(cx)}{3e(d+ex)^3} + \frac{bc^3(3c^2d^2-e^2)\log(d+ex)}{3(c^2d^2+e^2)^3} - \frac{bc^3(3c^2d^2-e^2)\log(1+c^2x^2)}{6(c^2d^2+e^2)^3}$$

```
[Out] -1/6*b*c/(c^2*d^2+e^2)/(e*x+d)^2-2/3*b*c^3*d/(c^2*d^2+e^2)^2/(e*x+d)+1/3*b*
c^4*d*(c^2*d^2-3*e^2)*arctan(c*x)/e/(c^2*d^2+e^2)^3+1/3*(-a-b*arctan(c*x))/
e/(e*x+d)^3+1/3*b*c^3*(3*c^2*d^2-e^2)*ln(e*x+d)/(c^2*d^2+e^2)^3-1/6*b*c^3*(
3*c^2*d^2-e^2)*ln(c^2*x^2+1)/(c^2*d^2+e^2)^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4972, 724, 815, 649, 209, 266}

$$\int \frac{a+b\arctan(cx)}{(d+ex)^4}\,dx = -\frac{a+b\arctan(cx)}{3e(d+ex)^3} + \frac{bc^4d\arctan(cx)(c^2d^2-3e^2)}{3e(c^2d^2+e^2)^3} - \frac{bc}{6(c^2d^2+e^2)(d+ex)^2} - \frac{bc^3(3c^2d^2-e^2)\log(c^2x^2+1)}{6(c^2d^2+e^2)^3} - \frac{2bc^3d}{3(c^2d^2+e^2)^2(d+ex)} + \frac{bc^3(3c^2d^2-e^2)\log(d+ex)}{3(c^2d^2+e^2)^3}$$

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x)^4,x]
```

```
[Out] -1/6*(b*c)/((c^2*d^2 + e^2)*(d + e*x)^2) - (2*b*c^3*d)/(3*(c^2*d^2 + e^2)^2
*(d + e*x)) + (b*c^4*d*(c^2*d^2 - 3*e^2)*ArcTan[c*x])/(3*e*(c^2*d^2 + e^2)^
3) - (a + b*ArcTan[c*x])/(3*e*(d + e*x)^3) + (b*c^3*(3*c^2*d^2 - e^2)*Log[d
 + e*x])/(3*(c^2*d^2 + e^2)^3) - (b*c^3*(3*c^2*d^2 - e^2)*Log[1 + c^2*x^2])
/(6*(c^2*d^2 + e^2)^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\arctan(cx)}{3e(d+ex)^3}+\frac{(bc)\int\frac{1}{(d+ex)^3(1+c^2x^2)}\,dx}{3e}$$

$$= -\frac{bc}{6(c^2d^2+e^2)(d+ex)^2} - \frac{a+b\arctan(cx)}{3e(d+ex)^3} + \frac{(bc^3)\int \frac{d-ex}{(d+ex)^2(1+c^2x^2)}\,dx}{3e(c^2d^2+e^2)}$$

$$= -\frac{bc}{6(c^2d^2+e^2)(d+ex)^2} - \frac{a+b\arctan(cx)}{3e(d+ex)^3} + \frac{(bc^3)\int \left(\frac{2de^2}{(c^2d^2+e^2)(d+ex)^2} - \frac{e^2(-3c^2d^2+e^2)}{(c^2d^2+e^2)^2(d+ex)} + \frac{c^2d(c^2d^2-3e^2)-c^2e(3c^2d^2-e^2)x}{(c^2d^2+e^2)^2(1+c^2x^2)}\right)dx}{3e(c^2d^2+e^2)}$$

$$= -\frac{bc}{6(c^2d^2+e^2)(d+ex)^2} - \frac{2bc^3d}{3(c^2d^2+e^2)^2(d+ex)} - \frac{a+b\arctan(cx)}{3e(d+ex)^3} + \frac{bc^3(3c^2d^2-e^2)\log(d+ex)}{3(c^2d^2+e^2)^3} + \frac{(bc^3)\int \frac{c^2d(c^2d^2-3e^2)-c^2e(3c^2d^2-e^2)x}{1+c^2x^2}\,dx}{3e(c^2d^2+e^2)^3}$$

$$= -\frac{bc}{6(c^2d^2+e^2)(d+ex)^2} - \frac{2bc^3d}{3(c^2d^2+e^2)^2(d+ex)} - \frac{a+b\arctan(cx)}{3e(d+ex)^3} + \frac{bc^3(3c^2d^2-e^2)\log(d+ex)}{3(c^2d^2+e^2)^3} + \frac{(bc^5d(c^2d^2-3e^2))\int \frac{1}{1+c^2x^2}\,dx}{3e(c^2d^2+e^2)^3} - \frac{(bc^5(3c^2d^2-e^2))\int \frac{x}{1+c^2x^2}\,dx}{3(c^2d^2+e^2)^3}$$

$$= -\frac{bc}{6(c^2d^2+e^2)(d+ex)^2} - \frac{2bc^3d}{3(c^2d^2+e^2)^2(d+ex)} + \frac{bc^4d(c^2d^2-3e^2)\arctan(cx)}{3e(c^2d^2+e^2)^3} - \frac{a+b\arctan(cx)}{3e(d+ex)^3} + \frac{bc^3(3c^2d^2-e^2)\log(d+ex)}{3(c^2d^2+e^2)^3} - \frac{bc^3(3c^2d^2-e^2)\log(1+c^2x^2)}{6(c^2d^2+e^2)^3}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.23

$$\int \frac{a+b\arctan(cx)}{(d+ex)^4}\,dx = -\frac{2(a+b\arctan(cx)) + \frac{bc(d+ex)\left(e(c^2d^2+e^2)^2+4c^2de(c^2d^2+e^2)(d+ex)-c^2\left(c^2d^2\left(\sqrt{-c^2}d-3e\right)+e^2\left(-3\sqrt{-c^2}d+e\right)\right)(d+ex)^2\log\left(1-\sqrt{\ldots}\right.}{(c^2d^{\ldots}}}{6e(d+ex)^3}$$

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x)^4,x]
```

```
[Out] -1/6*(2*(a + b*ArcTan[c*x]) + (b*c*(d + e*x)*(e*(c^2*d^2 + e^2)^2 + 4*c^2*d
*e*(c^2*d^2 + e^2)*(d + e*x) - c^2*(c^2*d^2*(Sqrt[-c^2]*d - 3*e) + e^2*(-3*
Sqrt[-c^2]*d + e))*(d + e*x)^2*Log[1 - Sqrt[-c^2]*x] - c^2*(e^2*(3*Sqrt[-c^
2]*d + e) - c^2*d^2*(Sqrt[-c^2]*d + 3*e))*(d + e*x)^2*Log[1 + Sqrt[-c^2]*x]
 - 2*c^2*e*(3*c^2*d^2 - e^2)*(d + e*x)^2*Log[d + e*x]))/(c^2*d^2 + e^2)^3)/
(e*(d + e*x)^3)
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

method	result
parts	$-\frac{a}{3(ex+d)^3 e}+\frac{b\left(-\frac{c^4\arctan(cx)}{3(cex+cd)^3 e}+\frac{c^4\left(-\frac{e}{2(c^2d^2+e^2)(cex+cd)^2}+\frac{e(3c^2d^2-e^2)\ln(cex+cd)}{(c^2d^2+e^2)^3}-\frac{2ecd}{(c^2d^2+e^2)^2(cex+cd)}+\frac{(-3c^2 \ldots}{\ldots}\right.}{3e}\right.}{c}$
derivativedivides	$\frac{-\frac{a c^4}{3(cex+cd)^3 e}+b c^4\left(-\frac{\arctan(cx)}{3(cex+cd)^3 e}+\frac{-\frac{e}{2(c^2d^2+e^2)(cex+cd)^2}+\frac{e(3c^2d^2-e^2)\ln(cex+cd)}{(c^2d^2+e^2)^3}-\frac{2ecd}{(c^2d^2+e^2)^2(cex+cd)}+\frac{(-3c^2d^2e- \ldots}{\ldots}}{3e}\right.}{c}$
default	$\frac{-\frac{a c^4}{3(cex+cd)^3 e}+b c^4\left(-\frac{\arctan(cx)}{3(cex+cd)^3 e}+\frac{-\frac{e}{2(c^2d^2+e^2)(cex+cd)^2}+\frac{e(3c^2d^2-e^2)\ln(cex+cd)}{(c^2d^2+e^2)^3}-\frac{2ecd}{(c^2d^2+e^2)^2(cex+cd)}+\frac{(-3c^2d^2e- \ldots}{\ldots}}{3e}\right.}{c}$
parallelrisch	$-\frac{5b c^7 d^5 e^3+6b c^5 d^3 e^5+b c^3 d e^7+6\ln(ex+d)x^2 b c^5 d e^7-3\ln(c^2x^2+1)x b c^5 d^2 e^6+6\ln(ex+d)x b c^5 d^2 e^6-2x^3\arctan(cx)b \ldots}{\ldots}$
risch	Expression too large to display

```
[In] int((a+b*arctan(c*x))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/(e*x+d)^3/e+b/c*(-1/3*c^4/(c*e*x+c*d)^3/e*arctan(c*x)+1/3*c^4/e*(-1/
2*e/(c^2*d^2+e^2)/(c*e*x+c*d)^2+e*(3*c^2*d^2-e^2)/(c^2*d^2+e^2)^3*ln(c*e*x+
c*d)-2*e*c*d/(c^2*d^2+e^2)^2/(c*e*x+c*d)+1/(c^2*d^2+e^2)^3*(1/2*(-3*c^2*d^2
*e+e^3)*ln(c^2*x^2+1)+(c^3*d^3-3*c*d*e^2)*arctan(c*x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(194) = 388.

Time = 0.64 (sec) , antiderivative size = 642, normalized size of antiderivative = 3.12

$$\int \frac{a+b\arctan(cx)}{(d+ex)^4}\,dx =$$
$$-\frac{2\,ac^6d^6+5\,bc^5d^5e+6\,ac^4d^4e^2+6\,bc^3d^3e^3+6\,ac^2d^2e^4+bcde^5+2\,ae^6+4\,(bc^5d^3e^3+bc^3de^5)x^2+(9\,bc^5 \ldots}{\ldots}$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^6*d^6 + 5*b*c^5*d^5*e + 6*a*c^4*d^4*e^2 + 6*b*c^3*d^3*e^3 + 6*a
*c^2*d^2*e^4 + b*c*d*e^5 + 2*a*e^6 + 4*(b*c^5*d^3*e^3 + b*c^3*d*e^5)*x^2 +
```

```
(9*b*c^5*d^4*e^2 + 10*b*c^3*d^2*e^4 + b*c*e^6)*x + 2*(6*b*c^4*d^4*e^2 + 3*b
*c^2*d^2*e^4 + b*e^6 - (b*c^6*d^3*e^3 - 3*b*c^4*d*e^5)*x^3 - 3*(b*c^6*d^4*e
^2 - 3*b*c^4*d^2*e^4)*x^2 - 3*(b*c^6*d^5*e - 3*b*c^4*d^3*e^3)*x)*arctan(c*x
) + (3*b*c^5*d^5*e - b*c^3*d^3*e^3 + (3*b*c^5*d^2*e^4 - b*c^3*e^6)*x^3 + 3*
(3*b*c^5*d^3*e^3 - b*c^3*d*e^5)*x^2 + 3*(3*b*c^5*d^4*e^2 - b*c^3*d^2*e^4)*x
)*log(c^2*x^2 + 1) - 2*(3*b*c^5*d^5*e - b*c^3*d^3*e^3 + (3*b*c^5*d^2*e^4 -
b*c^3*e^6)*x^3 + 3*(3*b*c^5*d^3*e^3 - b*c^3*d*e^5)*x^2 + 3*(3*b*c^5*d^4*e^2
 - b*c^3*d^2*e^4)*x)*log(e*x + d))/(c^6*d^9*e + 3*c^4*d^7*e^3 + 3*c^2*d^5*e
^5 + d^3*e^7 + (c^6*d^6*e^4 + 3*c^4*d^4*e^6 + 3*c^2*d^2*e^8 + e^10)*x^3 + 3
*(c^6*d^7*e^3 + 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 + d*e^9)*x^2 + 3*(c^6*d^8*e^2
 + 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 + d^2*e^8)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.69 (sec) , antiderivative size = 9202, normalized size of antiderivative = 44.67

$$\int \frac{a+b\arctan(cx)}{(d+ex)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*atan(c*x))/(e*x+d)**4,x)
```

```
[Out] Piecewise((a*x/d**4, Eq(c, 0) & Eq(e, 0)), ((a*x + b*x*atan(c*x) - b*log(x*
*2 + c**(-2))/(2*c))/d**4, Eq(e, 0)), (-a/(3*d**3*e + 9*d**2*e**2*x + 9*d*e
**3*x**2 + 3*e**4*x**3), Eq(c, 0)), (-24*a*d**3/(72*d**6*e + 216*d**5*e**2*
x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 21*I*b*d**3*atanh(e*x/d)/(72*
d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 10*I*b
*d**3/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3
) - 9*I*b*d**2*e*x*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**
3*x**2 + 72*d**3*e**4*x**3) + 9*I*b*d**2*e*x/(72*d**6*e + 216*d**5*e**2*x +
 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 9*I*b*d*e**2*x**2*atanh(e*x/d)/(
72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 3*I
*b*d*e**2*x**2/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*
e**4*x**3) - 3*I*b*e**3*x**3*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 21
6*d**4*e**3*x**2 + 72*d**3*e**4*x**3), Eq(c, -I*e/d)), (-24*a*d**3/(72*d**6
*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 21*I*b*d**
3*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*
e**4*x**3) - 10*I*b*d**3/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2
+ 72*d**3*e**4*x**3) + 9*I*b*d**2*e*x*atanh(e*x/d)/(72*d**6*e + 216*d**5*e*
*2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 9*I*b*d**2*e*x/(72*d**6*e
+ 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 9*I*b*d*e**2*
x**2*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d*
*3*e**4*x**3) - 3*I*b*d*e**2*x**2/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e
**3*x**2 + 72*d**3*e**4*x**3) + 3*I*b*e**3*x**3*atanh(e*x/d)/(72*d**6*e + 2
16*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3), Eq(c, I*e/d)), (-
```

```
2*a*c**6*d**6/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2
 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**
4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**
4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 +
18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 6*a*c**4*d**4*e**2/(6*c**
6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*
x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 1
8*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d
**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*
d*e**9*x**2 + 6*e**10*x**3) - 6*a*c**2*d**2*e**4/(6*c**6*d**9*e + 18*c**6*d
**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*
e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**
3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c
**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**1
0*x**3) - 2*a*e**6/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3
*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 5
4*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**
2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e*
*7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + 6*b*c**6*d**5*e*x*at
an(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c
**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5
*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6
*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**
2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + 6*b*c**6*d**4*e**2*x**2*atan(c*
x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d
**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5
*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x +
54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**
8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + 2*b*c**6*d**3*e**3*x**3*atan(c*x)/(6
*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e
**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2
 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c*
*2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x +
 18*d*e**9*x**2 + 6*e**10*x**3) - 3*b*c**5*d**5*e*log(x**2 + c**(-2))/(6*c*
*6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4
*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 +
18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*
d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18
*d*e**9*x**2 + 6*e**10*x**3) + 6*b*c**5*d**5*e*log(d/e + x)/(6*c**6*d**9*e
+ 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18
*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d*
*4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*
x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x*
*2 + 6*e**10*x**3) - 5*b*c**5*d**5*e/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x +
 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c*
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*4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*
d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**
8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 9*
b*c**5*d**4*e**2*x*log(x**2 + c**(-2))/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x
 + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*
c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**
2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e
**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) +
18*b*c**5*d**4*e**2*x*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 1
8*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4
*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d*
*5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*
x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 9*b*
c**5*d**4*e**2*x/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x
**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*
c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*
d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7
 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 9*b*c**5*d**3*e**3*x**
2*log(x**2 + c**(-2))/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e
**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x
+ 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*
c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3
*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + 18*b*c**5*d**3*e*
*3*x**2*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e*
*3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x +
 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c
**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*
e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 4*b*c**5*d**3*e**3
*x**2/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**
6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e
**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x
 + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*
e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 3*b*c**5*d**2*e**4*x**3*log(x**2
+ c**(-2))/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 +
6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d
**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e
**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*
d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + 6*b*c**5*d**2*e**4*x**3*log(
d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*
c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**
5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**
6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d*
*2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 12*b*c**4*d**4*e**2*atan(c*x)/
(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6
*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x*
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*2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*
c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x
 + 18*d*e**9*x**2 + 6*e**10*x**3) - 18*b*c**4*d**3*e**3*x*atan(c*x)/(6*c**6
*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x
**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18
*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d*
*3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d
*e**9*x**2 + 6*e**10*x**3) - 18*b*c**4*d**2*e**4*x**2*atan(c*x)/(6*c**6*d**
9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3
+ 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**
4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e
**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**
9*x**2 + 6*e**10*x**3) - 6*b*c**4*d*e**5*x**3*atan(c*x)/(6*c**6*d**9*e + 18
*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**
4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e
**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2
 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 +
 6*e**10*x**3) + b*c**3*d**3*e**3*log(x**2 + c**(-2))/(6*c**6*d**9*e + 18*c
**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*
d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**
6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 +
 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6
*e**10*x**3) - 2*b*c**3*d**3*e**3*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*d**
8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e*
*3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3
+ 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**
2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*
x**3) - 6*b*c**3*d**3*e**3/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d
**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e*
*4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5
+ 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6
*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + 3*b*c**3*d**
2*e**4*x*log(x**2 + c**(-2))/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6
*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*
e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**
5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 +
 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 6*b*c**3*d
**2*e**4*x*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7
*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*
x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 5
4*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d*
*3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 10*b*c**3*d**2*
e**4*x/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c*
*6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*
e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*
```

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x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2
*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + 3*b*c**3*d*e**5*x**2*log(x**2 +
c**(-2))/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*
c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**
5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**
6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d*
*2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 6*b*c**3*d*e**5*x**2*log(d/e +
 x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*
d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**
5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x +
 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e*
*8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 4*b*c**3*d*e**5*x**2/(6*c**6*d**9*e
 + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 1
8*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d
**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7
*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x
**2 + 6*e**10*x**3) + b*c**3*e**6*x**3*log(x**2 + c**(-2))/(6*c**6*d**9*e +
 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*
c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**
4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x
**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**
2 + 6*e**10*x**3) - 2*b*c**3*e**6*x**3*log(d/e + x)/(6*c**6*d**9*e + 18*c**
6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d*
*7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*
x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 1
8*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e
**10*x**3) - 6*b*c**2*d**2*e**4*atan(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**
2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 +
54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*
c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**
2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3)
 - b*c*d*e**5/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2
 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**
4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**
4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 +
18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - b*c*e**6*x/(6*c**6*d**9*e
 + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 1
8*c**4*d**7*e**3 + 54*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d
**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7
*x**2 + 18*c**2*d**2*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x
**2 + 6*e**10*x**3) - 2*b*e**6*atan(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2
*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 + 18*c**4*d**7*e**3 + 5
4*c**4*d**6*e**4*x + 54*c**4*d**5*e**5*x**2 + 18*c**4*d**4*e**6*x**3 + 18*c
**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2
*e**8*x**3 + 6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3),
```

```
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.82

$$\int \frac{a+b\arctan(cx)}{(d+ex)^4}\,dx = -\frac{1}{6}\left(c\left(\frac{(3\,c^4d^2-c^2e^2)\log\left(c^2x^2+1\right)}{c^6d^6+3\,c^4d^4e^2+3\,c^2d^2e^4+e^6}-\frac{2\,(3\,c^4d^2-c^2e^2)\log\left(ex+d\right)}{c^6d^6+3\,c^4d^4e^2+3\,c^2d^2e^4+e^6}+\frac{}{c^4d^6+2\,c^2d^4e^2+d^2e^4+(c^4 \cdots}\right.\right. - \frac{a}{3\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)}$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*((3*c^4*d^2 - c^2*e^2)*log(c^2*x^2 + 1)/(c^6*d^6 + 3*c^4*d^4*e^2 +
3*c^2*d^2*e^4 + e^6) - 2*(3*c^4*d^2 - c^2*e^2)*log(e*x + d)/(c^6*d^6 + 3*c^
4*d^4*e^2 + 3*c^2*d^2*e^4 + e^6) + (4*c^2*d*e*x + 5*c^2*d^2 + e^2)/(c^4*d^6
 + 2*c^2*d^4*e^2 + d^2*e^4 + (c^4*d^4*e^2 + 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c
^4*d^5*e + 2*c^2*d^3*e^3 + d*e^5)*x) - 2*(c^6*d^3 - 3*c^4*d*e^2)*arctan(c*x
)/((c^6*d^6*e + 3*c^4*d^4*e^3 + 3*c^2*d^2*e^5 + e^7)*c)) + 2*arctan(c*x)/(e
^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x
^2 + 3*d^2*e^2*x + d^3*e)
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex)^4}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex+d\right)^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex)^4}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{\left(d+e\,x\right)^4}\,dx$$

```
[In] int((a + b*atan(c*x))/(d + e*x)^4,x)
```

```
[Out] int((a + b*atan(c*x))/(d + e*x)^4, x)
```

3.9 $\int (d+ex)^3(a+b\arctan(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 376

$$\begin{aligned}\int (d+ex)^3(a+b\arctan(cx))^2\,dx &= \frac{b^2de^2x}{c^2}-\frac{abe(6c^2d^2-e^2)\,x}{2c^3}+\frac{b^2e^3x^2}{12c^2}\\&-\frac{b^2de^2\arctan(cx)}{c^3}-\frac{b^2e(6c^2d^2-e^2)\,x\arctan(cx)}{2c^3}\\&-\frac{bde^2x^2(a+b\arctan(cx))}{c}-\frac{be^3x^3(a+b\arctan(cx))}{6c}\\&+\frac{id(cd-e)(cd+e)(a+b\arctan(cx))^2}{c^3}\\&-\frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^2}{4c^4e}\\&+\frac{(d+ex)^4(a+b\arctan(cx))^2}{4e}\\&+\frac{2bd(cd-e)(cd+e)(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3}\\&-\frac{b^2e^3\log(1+c^2x^2)}{12c^4}+\frac{b^2e(6c^2d^2-e^2)\log(1+c^2x^2)}{4c^4}\\&+\frac{ib^2d(cd-e)(cd+e)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3}\end{aligned}$$

```
[Out] b^2*d*e^2*x/c^2-1/2*a*b*e*(6*c^2*d^2-e^2)*x/c^3+1/12*b^2*e^3*x^2/c^2-b^2*d*
e^2*arctan(c*x)/c^3-1/2*b^2*e*(6*c^2*d^2-e^2)*x*arctan(c*x)/c^3-b*d*e^2*x^2
*(a+b*arctan(c*x))/c-1/6*b*e^3*x^3*(a+b*arctan(c*x))/c+I*d*(c*d-e)*(c*d+e)*
(a+b*arctan(c*x))^2/c^3-1/4*(c^4*d^4-6*c^2*d^2*e^2+e^4)*(a+b*arctan(c*x))^2
/c^4/e+1/4*(e*x+d)^4*(a+b*arctan(c*x))^2/e+2*b*d*(c*d-e)*(c*d+e)*(a+b*arcta
n(c*x))*ln(2/(1+I*c*x))/c^3-1/12*b^2*e^3*ln(c^2*x^2+1)/c^4+1/4*b^2*e*(6*c^2
*d^2-e^2)*ln(c^2*x^2+1)/c^4+I*b^2*d*(c*d-e)*(c*d+e)*polylog(2,1-2/(1+I*c*x)
)/c^3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {4974, 4930, 266, 4946, 327, 209, 272, 45, 5104, 5004, 5040, 4964, 2449, 2352}

$$\int (d+ex)^3(a+b\arctan(cx))^2\,dx = \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^2}{c^3} + \frac{2bd(cd-e)(cd+e)\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^3} - \frac{(c^4d^4-6c^2d^2e^2+e^4)(a+b\arctan(cx))^2}{4c^4e} - \frac{bde^2x^2(a+b\arctan(cx))}{c} + \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} - \frac{be^3x^3(a+b\arctan(cx))}{6c} - \frac{abex(6c^2d^2-e^2)}{2c^3} - \frac{b^2de^2\arctan(cx)}{c^3} - \frac{b^2ex\arctan(cx)(6c^2d^2-e^2)}{2c^3} + \frac{ib^2d(cd-e)(cd+e)\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^3} + \frac{b^2de^2x}{c^2} + \frac{b^2e^3x^2}{12c^2} + \frac{b^2e(6c^2d^2-e^2)\log(c^2x^2+1)}{4c^4} - \frac{b^2e^3\log(c^2x^2+1)}{12c^4}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*d*e^2*x)/c^2 - (a*b*e*(6*c^2*d^2 - e^2)*x)/(2*c^3) + (b^2*e^3*x^2)/(12
*c^2) - (b^2*d*e^2*ArcTan[c*x])/c^3 - (b^2*e*(6*c^2*d^2 - e^2)*x*ArcTan[c*x
])/(2*c^3) - (b*d*e^2*x^2*(a + b*ArcTan[c*x]))/c - (b*e^3*x^3*(a + b*ArcTan
[c*x]))/(6*c) + (I*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2)/c^3 - ((c^4
*d^4 - 6*c^2*d^2*e^2 + e^4)*(a + b*ArcTan[c*x])^2)/(4*c^4*e) + ((d + e*x)^4
*(a + b*ArcTan[c*x])^2)/(4*e) + (2*b*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*
x])*Log[2/(1 + I*c*x)])/c^3 - (b^2*e^3*Log[1 + c^2*x^2])/(12*c^4) + (b^2*e*
(6*c^2*d^2 - e^2)*Log[1 + c^2*x^2])/(4*c^4) + (I*b^2*d*(c*d - e)*(c*d + e)*
PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} - \frac{(bc)\int\left(\frac{e^2(6c^2d^2-e^2)(a+b\arctan(cx))}{c^4} + \frac{4de^3x(a+b\arctan(cx))}{c^2} + \frac{e^4x^2(a+b\arctan(cx))}{c^2} + \frac{(c^4d^4-6c^2d^2e^2+e^4+4c^2d(cd-e)e(cd+e)}{c^4(1+c^2x^2)}\right.}{2e}$$

$$
\begin{aligned}
&= \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} - \frac{b\int \frac{(c^4d^4-6c^2d^2e^2+e^4+4c^2d(cd-e)e(cd+e)x)(a+b\arctan(cx))}{1+c^2x^2}\,dx}{2c^3e} \\
&\quad - \frac{(2bde^2)\int x(a+b\arctan(cx))\,dx}{c} - \frac{(be^3)\int x^2(a+b\arctan(cx))\,dx}{2c} \\
&\quad - \frac{(be(6c^2d^2-e^2))\int (a+b\arctan(cx))\,dx}{2c^3} \\
&= -\frac{abe(6c^2d^2-e^2)\,x}{2c^3} - \frac{bde^2x^2(a+b\arctan(cx))}{c} \\
&\quad - \frac{be^3x^3(a+b\arctan(cx))}{6c} + \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} \\
&\quad - \frac{b\int \left(\frac{c^4d^4\left(1+\frac{-6c^2d^2e^2+e^4}{c^4d^4}\right)(a+b\arctan(cx))}{1+c^2x^2} + \frac{4c^2d(cd-e)e(cd+e)x(a+b\arctan(cx))}{1+c^2x^2}\right)dx}{2c^3e} \\
&\quad + (b^2de^2)\int \frac{x^2}{1+c^2x^2}\,dx + \frac{1}{6}(b^2e^3)\int \frac{x^3}{1+c^2x^2}\,dx \\
&\quad - \frac{(b^2e(6c^2d^2-e^2))\int \arctan(cx)\,dx}{2c^3} \\
&= \frac{b^2de^2x}{c^2} - \frac{abe(6c^2d^2-e^2)\,x}{2c^3} - \frac{b^2e(6c^2d^2-e^2)\,x\arctan(cx)}{2c^3} \\
&\quad - \frac{bde^2x^2(a+b\arctan(cx))}{c} - \frac{be^3x^3(a+b\arctan(cx))}{6c} + \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} \\
&\quad - \frac{(b^2de^2)\int \frac{1}{1+c^2x^2}\,dx}{c^2} + \frac{1}{12}(b^2e^3)\,\mathrm{Subst}\left(\int \frac{x}{1+c^2x}\,dx, x, x^2\right) \\
&\quad - \frac{(2bd(cd-e)(cd+e))\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{c} + \frac{(b^2e(6c^2d^2-e^2))\int \frac{x}{1+c^2x^2}\,dx}{2c^2} \\
&\quad - \frac{(b(c^4d^4-6c^2d^2e^2+e^4))\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{2c^3e} \\
&= \frac{b^2de^2x}{c^2} - \frac{abe(6c^2d^2-e^2)\,x}{2c^3} - \frac{b^2de^2\arctan(cx)}{c^3} \\
&\quad - \frac{b^2e(6c^2d^2-e^2)\,x\arctan(cx)}{2c^3} - \frac{bde^2x^2(a+b\arctan(cx))}{c} \\
&\quad - \frac{be^3x^3(a+b\arctan(cx))}{6c} + \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^2}{c^3} \\
&\quad - \frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^2}{4c^4e} \\
&\quad + \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} + \frac{b^2e(6c^2d^2-e^2)\log(1+c^2x^2)}{4c^4} \\
&\quad + \frac{1}{12}(b^2e^3)\,\mathrm{Subst}\left(\int \left(\frac{1}{c^2} - \frac{1}{c^2(1+c^2x)}\right)dx, x, x^2\right) \\
&\quad + \frac{(2bd(cd-e)(cd+e))\int \frac{a+b\arctan(cx)}{i-cx}\,dx}{c^2}
\end{aligned}
$$

$$= \frac{b^2de^2x}{c^2} - \frac{abe(6c^2d^2-e^2)\,x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\arctan(cx)}{c^3} - \frac{b^2e(6c^2d^2-e^2)\,x\arctan(cx)}{2c^3} - \frac{bde^2x^2(a+b\arctan(cx))}{c} - \frac{be^3x^3(a+b\arctan(cx))}{6c} + \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^2}{c^3} - \frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^2}{4c^4e} + \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} + \frac{2bd(cd-e)(cd+e)(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{b^2e^3\log(1+c^2x^2)}{12c^4} + \frac{b^2e(6c^2d^2-e^2)\log(1+c^2x^2)}{4c^4} - \frac{(2b^2d(cd-e)(cd+e))\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2}$$

$$= \frac{b^2de^2x}{c^2} - \frac{abe(6c^2d^2-e^2)\,x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\arctan(cx)}{c^3} - \frac{b^2e(6c^2d^2-e^2)\,x\arctan(cx)}{2c^3} - \frac{bde^2x^2(a+b\arctan(cx))}{c} - \frac{be^3x^3(a+b\arctan(cx))}{6c} + \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^2}{c^3} - \frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^2}{4c^4e} + \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} + \frac{2bd(cd-e)(cd+e)(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{b^2e^3\log(1+c^2x^2)}{12c^4} + \frac{b^2e(6c^2d^2-e^2)\log(1+c^2x^2)}{4c^4} + \frac{(2ib^2d(cd-e)(cd+e))\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{c^3}$$

$$= \frac{b^2de^2x}{c^2} - \frac{abe(6c^2d^2-e^2)\,x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\arctan(cx)}{c^3} - \frac{b^2e(6c^2d^2-e^2)\,x\arctan(cx)}{2c^3} - \frac{bde^2x^2(a+b\arctan(cx))}{c} - \frac{be^3x^3(a+b\arctan(cx))}{6c} + \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^2}{c^3} - \frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^2}{4c^4e} + \frac{(d+ex)^4(a+b\arctan(cx))^2}{4e} + \frac{2bd(cd-e)(cd+e)(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{b^2e^3\log(1+c^2x^2)}{12c^4} + \frac{b^2e(6c^2d^2-e^2)\log(1+c^2x^2)}{4c^4} + \frac{ib^2d(cd-e)(cd+e)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.26

$$\int (d+ex)^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{b^2e^3+12a^2c^4d^3x-36abc^3d^2ex+12b^2c^2de^2x+6abce^3x+18a^2c^4d^2ex^2-12abc^3de^2x^2+b^2c^2e^3x^2+12a^2c^4}{}$$

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*e^3 + 12*a^2*c^4*d^3*x - 36*a*b*c^3*d^2*e*x + 12*b^2*c^2*d*e^2*x + 6*a
*b*c*e^3*x + 18*a^2*c^4*d^2*e*x^2 - 12*a*b*c^3*d*e^2*x^2 + b^2*c^2*e^3*x^2
+ 12*a^2*c^4*d*e^2*x^3 - 2*a*b*c^3*e^3*x^3 + 3*a^2*c^4*e^3*x^4 + 3*b^2*((-4
*I)*c^3*d^3 + 6*c^2*d^2*e + (4*I)*c*d*e^2 - e^3 + c^4*x*(4*d^3 + 6*d^2*e*x
+ 4*d*e^2*x^2 + e^3*x^3))*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-(b*c*e*(18*c^2*
d^2*x + e^2*x*(-3 + c^2*x^2) + 6*d*(e + c^2*e*x^2))) + 3*a*(6*c^2*d^2*e - e
^3 + c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)) + 12*b*c*d*(c^2*d^2
 - e^2)*Log[1 + E^((2*I)*ArcTan[c*x])]) - 12*a*b*c^3*d^3*Log[1 + c^2*x^2] +
 18*b^2*c^2*d^2*e*Log[1 + c^2*x^2] + 12*a*b*c*d*e^2*Log[1 + c^2*x^2] - 4*b^
2*e^3*Log[1 + c^2*x^2] - (12*I)*b^2*c*d*(c^2*d^2 - e^2)*PolyLog[2, -E^((2*I
)*ArcTan[c*x])])/(12*c^4)
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.77

method	result
parts	$\frac{a^2(ex+d)^4}{4e} + \frac{b^2\left(\frac{c\,e^3\arctan(cx)^2x^4}{4} + c\,e^2\arctan(cx)^2x^3d + \frac{3ce\arctan(cx)^2x^2d^2}{2} + \arctan(cx)^2cx\,d^3 + \frac{c\arctan(cx)^2d^4}{4e} - \frac{6\,a\cdots}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a^2(cex+cd)^4}{4c^3e} + \frac{b^2\left(\frac{\arctan(cx)^2c^4d^4}{4e} + \arctan(cx)^2c^4d^3x + \frac{3e\arctan(cx)^2c^4d^2x^2}{2} + e^2\arctan(cx)^2c^4d\,x^3 + \frac{e^3\arctan(cx)^2c^4x^4}{4} - \frac{6\arctan\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{a^2(cex+cd)^4}{4c^3e} + \frac{b^2\left(\frac{\arctan(cx)^2c^4d^4}{4e} + \arctan(cx)^2c^4d^3x + \frac{3e\arctan(cx)^2c^4d^2x^2}{2} + e^2\arctan(cx)^2c^4d\,x^3 + \frac{e^3\arctan(cx)^2c^4x^4}{4} - \frac{6\arctan\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

```
[In] int((e*x+d)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*(e*x+d)^4/e+b^2/c*(1/4*c*e^3*arctan(c*x)^2*x^4+c*e^2*arctan(c*x)^2*
x^3*d+3/2*c*e*arctan(c*x)^2*x^2*d^2+arctan(c*x)^2*c*x*d^3+1/4*c/e*arctan(c*
x)^2*d^4-1/2/c^3/e*(6*arctan(c*x)*c^3*d^2*e^2*x+2*arctan(c*x)*e^3*c^3*d*x^2
+1/3*arctan(c*x)*e^4*c^3*x^3-arctan(c*x)*e^4*c*x+2*arctan(c*x)*ln(c^2*x^2+1
)*c^3*d^3*e-2*arctan(c*x)*ln(c^2*x^2+1)*c*d*e^3+arctan(c*x)^2*c^4*d^4-6*arc
tan(c*x)^2*c^2*d^2*e^2+arctan(c*x)^2*e^4-1/12*(6*c^4*d^4-36*c^2*d^2*e^2+6*e
^4)*arctan(c*x)^2-1/3*e^2*(6*c^2*d*e*x+1/2*c^2*e^2*x^2+1/2*(18*c^2*d^2-4*e^
2)*ln(c^2*x^2+1)-6*e*arctan(c*x)*c*d)-2*c*d*e*(c^2*d^2-e^2)*(-1/2*I*(ln(c*x
-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln
(c*x-I)^2)+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln
(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2))))+2*a*b/c*(1/4*c*e^3*arctan(c*x)*x^4+c*e^
2*arctan(c*x)*x^3*d+3/2*c*e*arctan(c*x)*x^2*d^2+arctan(c*x)*c*x*d^3+1/4*c/e
*arctan(c*x)*d^4-1/4/c^3/e*(6*c^3*d^2*e^2*x+2*e^3*c^3*d*x^2+1/3*e^4*c^3*x^3
-c*e^4*x+1/2*(4*c^3*d^3*e-4*c*d*e^3)*ln(c^2*x^2+1)+(c^4*d^4-6*c^2*d^2*e^2+e
^4)*arctan(c*x)))
```

Fricas [F]

$$\int (d+ex)^3(a+b\arctan(cx))^2\,dx = \int (ex+d)^3(b\arctan(cx)+a)^2\,dx$$

[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")

[Out] integral(a^2*e^3*x^3 + 3*a^2*d*e^2*x^2 + 3*a^2*d^2*e*x + a^2*d^3 + (b^2*e^3*x^3 + 3*b^2*d*e^2*x^2 + 3*b^2*d^2*e*x + b^2*d^3)*arctan(c*x)^2 + 2*(a*b*e^3*x^3 + 3*a*b*d*e^2*x^2 + 3*a*b*d^2*e*x + a*b*d^3)*arctan(c*x), x)

Sympy [F]

$$\int (d+ex)^3(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(cx))^2\,(d+ex)^3\,dx$$

[In] integrate((e*x+d)**3*(a+b*atan(c*x))**2,x)

[Out] Integral((a + b*atan(c*x))**2*(d + e*x)**3, x)

Maxima [F]

$$\int (d+ex)^3(a+b\arctan(cx))^2\,dx = \int (ex+d)^3(b\arctan(cx)+a)^2\,dx$$

[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")

[Out] 1/4*a^2*e^3*x^4 + a^2*d*e^2*x^3 + 12*b^2*c^2*e^3*integrate(1/16*x^5*arctan(c*x)^2/(c^2*x^2 + 1), x) + b^2*c^2*e^3*integrate(1/16*x^5*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 36*b^2*c^2*d*e^2*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) + b^2*c^2*e^3*integrate(1/16*x^5*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*b^2*c^2*d*e^2*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 36*b^2*c^2*d^2*e*integrate(1/16*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + 4*b^2*c^2*d*e^2*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*b^2*c^2*d^2*e*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d^3*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 6*b^2*c^2*d^2*e*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + b^2*c^2*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 4*b^2*c^2*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3/2*a^2*d^2*e*x^2 + 1/4*b^2*d^3*arctan(c*x)^3/c - 2*b^2*c*e^3*integrate(1/16*x^4*arctan(c*x)/(c^2*x^2 + 1), x) - 8*b^2*c*d*e^2*integrate(1/16*x^3*arctan(c*x)/(c^2*x^2 + 1), x) - 12*b^2*c*d^2*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^2 + 1), x) - 8

```
*b^2*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + 3*(x^2*arctan(c
*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^2*e + (2*x^3*arctan(c*x) - c*(x^2/
c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d*e^2 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x
^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*e^3 + a^2*d^3*x + 12*b^2*e^3*integr
ate(1/16*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + b^2*e^3*integrate(1/16*x^3*l
og(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 36*b^2*d*e^2*integrate(1/16*x^2*arcta
n(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*d*e^2*integrate(1/16*x^2*log(c^2*x^2 + 1
)^2/(c^2*x^2 + 1), x) + 36*b^2*d^2*e*integrate(1/16*x*arctan(c*x)^2/(c^2*x^
2 + 1), x) + 3*b^2*d^2*e*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1),
 x) + b^2*d^3*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*
arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^3/c + 1/16*(b^2*e^3*x^4 + 4*b^2*d*e^2
*x^3 + 6*b^2*d^2*e*x^2 + 4*b^2*d^3*x)*arctan(c*x)^2 - 1/64*(b^2*e^3*x^4 + 4
*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2 + 4*b^2*d^3*x)*log(c^2*x^2 + 1)^2
```

Giac [F]

$$\int (d+ex)^3(a+b\arctan(cx))^2\,dx = \int (ex+d)^3(b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(c\,x))^2\,(d+e\,x)^3\,dx$$

```
[In] int((a + b*atan(c*x))^2*(d + e*x)^3,x)
```

```
[Out] int((a + b*atan(c*x))^2*(d + e*x)^3, x)
```

3.10 $\int (d+ex)^2(a+b\arctan(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 270

$$\begin{aligned}\int (d+ex)^2(a+b\arctan(cx))^2\,dx &= -\frac{2abdex}{c}+\frac{b^2e^2x}{3c^2}-\frac{b^2e^2\arctan(cx)}{3c^3} \\ &-\frac{2b^2dex\arctan(cx)}{c}-\frac{be^2x^2(a+b\arctan(cx))}{3c} \\ &+\frac{i(3c^2d^2-e^2)\,(a+b\arctan(cx))^2}{3c^3} \\ &-\frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^2}{3e} \\ &+\frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} \\ &+\frac{2b(3c^2d^2-e^2)\,(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} \\ &+\frac{b^2de\log(1+c^2x^2)}{c^2} \\ &+\frac{ib^2(3c^2d^2-e^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\end{aligned}$$

```
[Out] -2*a*b*d*e*x/c+1/3*b^2*e^2*x/c^2-1/3*b^2*e^2*arctan(c*x)/c^3-2*b^2*d*e*x*ar
ctan(c*x)/c-1/3*b*e^2*x^2*(a+b*arctan(c*x))/c+1/3*I*(3*c^2*d^2-e^2)*(a+b*ar
ctan(c*x))^2/c^3-1/3*d*(d^2-3*e^2/c^2)*(a+b*arctan(c*x))^2/e+1/3*(e*x+d)^3*
(a+b*arctan(c*x))^2/e+2/3*b*(3*c^2*d^2-e^2)*(a+b*arctan(c*x))*ln(2/(1+I*c*x
))/c^3+b^2*d*e*ln(c^2*x^2+1)/c^2+1/3*I*b^2*(3*c^2*d^2-e^2)*polylog(2,1-2/(1
+I*c*x))/c^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4974, 4930, 266, 4946, 327, 209, 5104, 5004, 5040, 4964, 2449, 2352}

$$\int (d+ex)^2(a+b\arctan(cx))^2\,dx = -\frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^2}{3e} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^2}{3c^3} + \frac{2b(3c^2d^2-e^2)\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3} + \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} - \frac{be^2x^2(a+b\arctan(cx))}{3c} - \frac{2abdex}{c} - \frac{b^2e^2\arctan(cx)}{3c^3} - \frac{2b^2dex\arctan(cx)}{c} + \frac{b^2de\log(c^2x^2+1)}{c^2} + \frac{b^2e^2x}{3c^2} + \frac{ib^2(3c^2d^2-e^2)\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-2*a*b*d*e*x)/c + (b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTan[c*x])/(3*c^3) - (2
*b^2*d*e*x*ArcTan[c*x])/c - (b*e^2*x^2*(a + b*ArcTan[c*x]))/(3*c) + ((I/3)*
(3*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])^2)/c^3 - (d*(d^2 - (3*e^2)/c^2)*(a +
b*ArcTan[c*x])^2)/(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x])^2)/(3*e) + (2*b*
(3*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (b^2*d*
e*Log[1 + c^2*x^2])/c^2 + ((I/3)*b^2*(3*c^2*d^2 - e^2)*PolyLog[2, 1 - 2/(1
+ I*c*x)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} - \frac{(2bc)\int\left(\frac{3de^2(a+b\arctan(cx))}{c^2}+\frac{e^3x(a+b\arctan(cx))}{c^2}+\frac{(c^2d^3-3de^2+e(3c^2d^2-e^2)x)(a+b\arctan(cx))}{c^2(1+c^2x^2)}\right)dx}{3e}\\
&= \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} - \frac{(2b)\int\frac{(c^2d^3-3de^2+e(3c^2d^2-e^2)x)(a+b\arctan(cx))}{1+c^2x^2}\,dx}{3ce}\\
&\quad - \frac{(2bde)\int(a+b\arctan(cx))\,dx}{c} - \frac{(2be^2)\int x(a+b\arctan(cx))\,dx}{3c}\\
&= -\frac{2abdex}{c} - \frac{be^2x^2(a+b\arctan(cx))}{3c} + \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e}\\
&\quad - \frac{(2b)\int\left(\frac{c^2d^3\left(1-\frac{3e^2}{c^2d^2}\right)(a+b\arctan(cx))}{1+c^2x^2}-\frac{e(-3c^2d^2+e^2)x(a+b\arctan(cx))}{1+c^2x^2}\right)dx}{3ce}\\
&\quad - \frac{(2b^2de)\int\arctan(cx)\,dx}{c} + \frac{1}{3}(b^2e^2)\int\frac{x^2}{1+c^2x^2}\,dx\\
&= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{2b^2dex\arctan(cx)}{c} - \frac{be^2x^2(a+b\arctan(cx))}{3c}\\
&\quad + \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} + (2b^2de)\int\frac{x}{1+c^2x^2}\,dx - \frac{(b^2e^2)\int\frac{1}{1+c^2x^2}\,dx}{3c^2}\\
&\quad - \frac{1}{3}\left(2bd\left(\frac{cd^2}{e}-\frac{3e}{c}\right)\right)\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx - \frac{(2b(3c^2d^2-e^2))\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{3c}
\end{aligned}
$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\arctan(cx)}{3c^3} - \frac{2b^2dex\arctan(cx)}{c} - \frac{be^2x^2(a+b\arctan(cx))}{3c} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^2}{3c^3} - \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^2}{3e} + \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} + \frac{b^2de\log(1+c^2x^2)}{c^2} + \frac{(2b(3c^2d^2-e^2))\int\frac{a+b\arctan(cx)}{i-cx}dx}{3c^2}$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\arctan(cx)}{3c^3} - \frac{2b^2dex\arctan(cx)}{c} - \frac{be^2x^2(a+b\arctan(cx))}{3c} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^2}{3c^3} - \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^2}{3e} + \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} + \frac{2b(3c^2d^2-e^2)(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} + \frac{b^2de\log(1+c^2x^2)}{c^2} - \frac{(2b^2(3c^2d^2-e^2))\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}dx}{3c^2}$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\arctan(cx)}{3c^3} - \frac{2b^2dex\arctan(cx)}{c} - \frac{be^2x^2(a+b\arctan(cx))}{3c} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^2}{3c^3} - \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^2}{3e} + \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} + \frac{2b(3c^2d^2-e^2)(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} + \frac{b^2de\log(1+c^2x^2)}{c^2} + \frac{(2ib^2(3c^2d^2-e^2))\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1+icx}\right)}{3c^3}$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\arctan(cx)}{3c^3} - \frac{2b^2dex\arctan(cx)}{c} - \frac{be^2x^2(a+b\arctan(cx))}{3c} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^2}{3c^3} - \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^2}{3e} + \frac{(d+ex)^3(a+b\arctan(cx))^2}{3e} + \frac{2b(3c^2d^2-e^2)(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} + \frac{b^2de\log(1+c^2x^2)}{c^2} + \frac{ib^2(3c^2d^2-e^2)\operatorname{PolyLog}\left(2, 1-\frac{2}{1+icx}\right)}{3c^3}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.16

$$\int (d+ex)^2(a+b\arctan(cx))^2\,dx$$
$$=\frac{3a^2c^3d^2x-6abc^2dex+b^2ce^2x+3a^2c^3dex^2-abc^2e^2x^2+a^2c^3e^2x^3+b^2(-3ic^2d^2+3cde+ie^2+c^3x(3d^2+$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (3*a^2*c^3*d^2*x - 6*a*b*c^2*d*e*x + b^2*c*e^2*x + 3*a^2*c^3*d*e*x^2 - a*b*
c^2*e^2*x^2 + a^2*c^3*e^2*x^3 + b^2*((-3*I)*c^2*d^2 + 3*c*d*e + I*e^2 + c^3
*x*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcTan[c*x]^2 + b*ArcTan[c*x]*(6*a*c*d*e -
b*e*(e + 6*c^2*d*x + c^2*e*x^2) + 2*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) + 2
*b*(3*c^2*d^2 - e^2)*Log[1 + E^((2*I)*ArcTan[c*x])]) - 3*a*b*c^2*d^2*Log[1
+ c^2*x^2] + 3*b^2*c*d*e*Log[1 + c^2*x^2] + a*b*e^2*Log[1 + c^2*x^2] - I*b^
2*(3*c^2*d^2 - e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(250) = 500.

Time = 2.08 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.86

method	result
parts	$\frac{a^2(ex+d)^3}{3e}+\frac{b^2\left(\frac{c\,e^2\arctan(cx)^2x^3}{3}+ce\arctan(cx)^2x^2d+\arctan(cx)^2cx\,d^2+\frac{c\arctan(cx)^2d^3}{3e}-\frac{2\left(3\arctan(cx)c^2d\,e^2x+\frac{\arctan}{}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2(cex+cd)^3}{3c^2e}+\frac{b^2\left(\frac{\arctan(cx)^2c^3d^3}{3e}+\arctan(cx)^2c^3d^2x+e\arctan(cx)^2c^3d\,x^2+\frac{e^2\arctan(cx)^2c^3x^3}{3}-\frac{2\left(3\arctan(cx)c^2d\,e^2x+\frac{\arctan(}{}\right.}{}\right.}{}}{}$
default	$\frac{\frac{a^2(cex+cd)^3}{3c^2e}+\frac{b^2\left(\frac{\arctan(cx)^2c^3d^3}{3e}+\arctan(cx)^2c^3d^2x+e\arctan(cx)^2c^3d\,x^2+\frac{e^2\arctan(cx)^2c^3x^3}{3}-\frac{2\left(3\arctan(cx)c^2d\,e^2x+\frac{\arctan(}{}\right.}{}\right.}{}}{}$
risch	$\frac{b^2e^2x}{3c^2}-\frac{2abdex}{c}+\frac{7b^2de\ln(c^2x^2+1)}{8c^2}-\frac{17b^2e^2\arctan(cx)}{36c^3}-\frac{e^2ba\,x^2}{3c}+x^2ed\,a^2+\frac{x^3e^2a^2}{3}+x\,d^2a^2+i\ln$

```
[In] int((e*x+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*(e*x+d)^3/e+b^2/c*(1/3*c*e^2*arctan(c*x)^2*x^3+c*e*arctan(c*x)^2*x^
2*d+arctan(c*x)^2*c*x*d^2+1/3*c/e*arctan(c*x)^2*d^3-2/3/c^2/e*(3*arctan(c*x
)*c^2*d*e^2*x+1/2*arctan(c*x)*e^3*c^2*x^2+3/2*arctan(c*x)*ln(c^2*x^2+1)*e*c
^2*d^2-1/2*arctan(c*x)*ln(c^2*x^2+1)*e^3+arctan(c*x)^2*c^3*d^3-3*arctan(c*x
)^2*c*d*e^2-1/2*e*(3*c^2*d^2-e^2)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1
```

```
/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)+1/2*I*(ln(c*x+I
)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x
+I)^2))-3/2*e^2*ln(c^2*x^2+1)*c*d+1/2*e^3*arctan(c*x)-1/2*c*x*e^3-1/2*d*c*(
c^2*d^2-3*e^2)*arctan(c*x)^2))+2/3*a*b*e^2*arctan(c*x)*x^3+2*a*b*e*arctan(c
*x)*x^2*d+2*a*b*arctan(c*x)*x*d^2-1/3/c*e^2*b*a*x^2-2*a*b*d*e*x/c-1/c*a*b*d
^2*ln(c^2*x^2+1)+1/3/c^3*e^2*b*a*ln(c^2*x^2+1)+2/c^2*d*e*b*a*arctan(c*x)
```

Fricas [F]

$$\int (d+ex)^2(a+b\arctan(cx))^2\,dx=\int (ex+d)^2(b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e*x +
 b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arctan(c*
x), x)
```

Sympy [F]

$$\int (d+ex)^2(a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}(cx))^2\,(d+ex)^2\,dx$$

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x)**2, x)
```

Maxima [F]

$$\int (d+ex)^2(a+b\arctan(cx))^2\,dx=\int (ex+d)^2(b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*e^2*x^3 + 36*b^2*c^2*e^2*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^2
+ 1), x) + 3*b^2*c^2*e^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1
), x) + 72*b^2*c^2*d*e*integrate(1/48*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) +
 4*b^2*c^2*e^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 6*b^
2*c^2*d*e*integrate(1/48*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 36*b^2*
c^2*d^2*integrate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d*e
*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*b^2*c^2*d^2*inte
grate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d^2*integr
```

```
ate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + a^2*d*e*x^2 + 1/4*b^2*d^2
*arctan(c*x)^3/c - 8*b^2*c*e^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2 + 1)
, x) - 24*b^2*c*d*e*integrate(1/48*x^2*arctan(c*x)/(c^2*x^2 + 1), x) - 24*b
^2*c*d^2*integrate(1/48*x*arctan(c*x)/(c^2*x^2 + 1), x) + 2*(x^2*arctan(c*x
) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d*e + 1/3*(2*x^3*arctan(c*x) - c*(x^2/
c^2 - log(c^2*x^2 + 1)/c^4))*a*b*e^2 + a^2*d^2*x + 36*b^2*e^2*integrate(1/4
8*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*e^2*integrate(1/48*x^2*log(c^
2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 72*b^2*d*e*integrate(1/48*x*arctan(c*x)^2/
(c^2*x^2 + 1), x) + 6*b^2*d*e*integrate(1/48*x*log(c^2*x^2 + 1)^2/(c^2*x^2
+ 1), x) + 3*b^2*d^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) +
(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^2/c + 1/12*(b^2*e^2*x^3 + 3*b^
2*d*e*x^2 + 3*b^2*d^2*x)*arctan(c*x)^2 - 1/48*(b^2*e^2*x^3 + 3*b^2*d*e*x^2
+ 3*b^2*d^2*x)*log(c^2*x^2 + 1)^2
```

Giac [F]

$$\int (d+ex)^2(a+b\arctan(cx))^2\,dx=\int (ex+d)^2(b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}(c\,x))^2\,(d+e\,x)^2\,dx$$

```
[In] int((a + b*atan(c*x))^2*(d + e*x)^2,x)
```

```
[Out] int((a + b*atan(c*x))^2*(d + e*x)^2, x)
```

3.11 $\int (d+ex)(a+b\arctan(cx))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 171

$$\begin{aligned}\int (d+ex)(a+b\arctan(cx))^2\,dx = & -\frac{abex}{c}-\frac{b^2ex\arctan(cx)}{c}+\frac{id(a+b\arctan(cx))^2}{c}\\ & -\frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^2}{2e}\\ & +\frac{(d+ex)^2(a+b\arctan(cx))^2}{2e}\\ & +\frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}\\ & +\frac{b^2e\log(1+c^2x^2)}{2c^2}+\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}\end{aligned}$$

```
[Out] -a*b*e*x/c-b^2*e*x*arctan(c*x)/c+I*d*(a+b*arctan(c*x))^2/c-1/2*(d^2-e^2/c^2
)*(a+b*arctan(c*x))^2/e+1/2*(e*x+d)^2*(a+b*arctan(c*x))^2/e+2*b*d*(a+b*arct
an(c*x))*ln(2/(1+I*c*x))/c+1/2*b^2*e*ln(c^2*x^2+1)/c^2+I*b^2*d*polylog(2,1-
2/(1+I*c*x))/c
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used

$= \{4974, 4930, 266, 5104, 5004, 5040, 4964, 2449, 2352\}$

$$\int (d+ex)(a+b\arctan(cx))^2\,dx = -\frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^2}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} + \frac{id(a+b\arctan(cx))^2}{c} + \frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c} - \frac{abex}{c} - \frac{b^2ex\arctan(cx)}{c} + \frac{b^2e\log(c^2x^2+1)}{2c^2} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*e*x)/c) - (b^2*e*x*ArcTan[c*x])/c + (I*d*(a + b*ArcTan[c*x])^2)/c -
((d^2 - e^2/c^2)*(a + b*ArcTan[c*x])^2)/(2*e) + ((d + e*x)^2*(a + b*ArcTan[
c*x])^2)/(2*e) + (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + (b^2*e*
Log[1 + c^2*x^2])/(2*c^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} - \frac{(bc)\int\left(\frac{e^2(a+b\arctan(cx))}{c^2}+\frac{(c^2d^2-e^2+2c^2dex)(a+b\arctan(cx))}{c^2(1+c^2x^2)}\right)dx}{e}\\ &= \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} - \frac{b\int\frac{(c^2d^2-e^2+2c^2dex)(a+b\arctan(cx))}{1+c^2x^2}dx}{ce} - \frac{(be)\int(a+b\arctan(cx))\,dx}{c}\end{aligned}$$

$$= -\frac{abex}{c} + \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} - \frac{b\int\left(\frac{c^2d^2\left(1-\frac{e^2}{c^2d^2}\right)(a+b\arctan(cx))}{1+c^2x^2} + \frac{2c^2dex(a+b\arctan(cx))}{1+c^2x^2}\right)dx}{ce} - \frac{(b^2e)\int\arctan(cx)\,dx}{c}$$

$$= -\frac{abex}{c} - \frac{b^2ex\arctan(cx)}{c} + \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} - (2bcd)\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx + (b^2e)\int\frac{x}{1+c^2x^2}\,dx - \frac{(b(cd-e)(cd+e))\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{ce}$$

$$= -\frac{abex}{c} - \frac{b^2ex\arctan(cx)}{c} + \frac{id(a+b\arctan(cx))^2}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^2}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} + \frac{b^2e\log(1+c^2x^2)}{2c^2} + (2bd)\int\frac{a+b\arctan(cx)}{i-cx}\,dx$$

$$= -\frac{abex}{c} - \frac{b^2ex\arctan(cx)}{c} + \frac{id(a+b\arctan(cx))^2}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^2}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} + \frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} + \frac{b^2e\log(1+c^2x^2)}{2c^2} - (2b^2d)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx$$

$$= -\frac{abex}{c} - \frac{b^2ex\arctan(cx)}{c} + \frac{id(a+b\arctan(cx))^2}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^2}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} + \frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} + \frac{b^2e\log(1+c^2x^2)}{2c^2} + \frac{(2ib^2d)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{c}$$

$$= -\frac{abex}{c} - \frac{b^2ex\arctan(cx)}{c} + \frac{id(a+b\arctan(cx))^2}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^2}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))^2}{2e} + \frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} + \frac{b^2e\log(1+c^2x^2)}{2c^2} + \frac{ib^2d\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int (d+ex)(a+b\arctan(cx))^2\,dx$$
$$=\frac{2a^2c^2dx-2abcex+a^2c^2ex^2+b^2(-i+cx)(2cd+ie+cex)\arctan(cx)^2+2b\arctan(cx)\left(-bcex+a(e+2\right.}{}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (2*a^2*c^2*d*x - 2*a*b*c*e*x + a^2*c^2*e*x^2 + b^2*(-I + c*x)*(2*c*d + I*e
+ c*e*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-(b*c*e*x) + a*(e + 2*c^2*d*x + c
^2*e*x^2) + 2*b*c*d*Log[1 + E^((2*I)*ArcTan[c*x])]) - 2*a*b*c*d*Log[1 + c^2
*x^2] + b^2*e*Log[1 + c^2*x^2] - (2*I)*b^2*c*d*PolyLog[2, -E^((2*I)*ArcTan[
c*x])])/(2*c^2)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.71

method	result
parts	$a^2\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b^2\left(\frac{\arctan(cx)^2c\,x^2e}{2}+\arctan(cx)^2cxd-\frac{\ln\left(c^2x^2+1\right)\arctan(cx)cd-\frac{\arctan(cx)^2e}{2}+\arctan(cx)ecx-\frac{e\ln}{}}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(\arctan(cx)^2d\,c^2x+\frac{\arctan(cx)^2e\,c^2x^2}{2}-\ln\left(c^2x^2+1\right)\arctan(cx)cd+\frac{\arctan(cx)^2e}{2}-\arctan(cx)ecx+\frac{e\ln\left(c^2\right.}{2}\right.}{}}{}$
default	$\frac{\frac{a^2\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(\arctan(cx)^2d\,c^2x+\frac{\arctan(cx)^2e\,c^2x^2}{2}-\ln\left(c^2x^2+1\right)\arctan(cx)cd+\frac{\arctan(cx)^2e}{2}-\arctan(cx)ecx+\frac{e\ln\left(c^2\right.}{2}\right.}{}}{}$
risch	$-\frac{abex}{c}+\frac{b^2e\ln(c^2x^2+1)}{4c^2}+a^2dx+\frac{a^2e\,x^2}{2}-\frac{e\,b^2\ln(-icx+1)^2}{8c^2}+\frac{e\,b^2\ln(-icx+1)}{2c^2}-\frac{e\,b^2\ln(-icx+1)^2x^2}{8}-$

```
[In] int((e*x+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*e*x^2+d*x)+b^2/c*(1/2*arctan(c*x)^2*c*x^2*e+arctan(c*x)^2*c*x*d-1/
c*(ln(c^2*x^2+1)*arctan(c*x)*c*d-1/2*arctan(c*x)^2*e+arctan(c*x)*e*c*x-1/2*
e*ln(c^2*x^2+1)-d*c*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-
ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1
)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2))))+a*b*
```

```
arctan(c*x)*x^2*e+2*a*b*arctan(c*x)*x*d-1/c*a*b*d*ln(c^2*x^2+1)-a*b*e*x/c+1
/c^2*e*b*a*arctan(c*x)
```

Fricas [F]

$$\int (d+ex)(a+b\arctan(cx))^2\,dx = \int (ex+d)(b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
[Out] integral(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctan(c*x)^2 + 2*(a*b*e*x + a
*b*d)*arctan(c*x), x)
```

Sympy [F]

$$\int (d+ex)(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(cx))^2\,(d+ex)\,dx$$

```
[In] integrate((e*x+d)*(a+b*atan(c*x))**2,x)
[Out] Integral((a + b*atan(c*x))**2*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex)(a+b\arctan(cx))^2\,dx = \int (ex+d)(b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
[Out] 12*b^2*c^2*e*integrate(1/16*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + b^2*c^2*e
*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d*int
egrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 2*b^2*c^2*e*integrate(1/1
6*x^3*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + b^2*c^2*d*integrate(1/16*x^2*log
(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 4*b^2*c^2*d*integrate(1/16*x^2*log(c^2*
x^2 + 1)/(c^2*x^2 + 1), x) + 1/2*a^2*e*x^2 + 1/4*b^2*d*arctan(c*x)^3/c - 4*
b^2*c*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^2 + 1), x) - 8*b^2*c*d*integr
ate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + (x^2*arctan(c*x) - c*(x/c^2 - ar
ctan(c*x)/c^3))*a*b*e + a^2*d*x + 12*b^2*e*integrate(1/16*x*arctan(c*x)^2/(
c^2*x^2 + 1), x) + b^2*e*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1),
 x) + b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*ar
ctan(c*x) - log(c^2*x^2 + 1))*a*b*d/c + 1/8*(b^2*e*x^2 + 2*b^2*d*x)*arctan(
c*x)^2 - 1/32*(b^2*e*x^2 + 2*b^2*d*x)*log(c^2*x^2 + 1)^2
```

Giac [F]

$$\int (d+ex)(a+b\arctan(cx))^2\,dx = \int (ex+d)(b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(c\,x))^2\,(d+e\,x)\,dx$$

```
[In] int((a + b*atan(c*x))^2*(d + e*x),x)
[Out] int((a + b*atan(c*x))^2*(d + e*x), x)
```

3.12 $\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 223

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = &-\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e}\\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}\\ &+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e}\\ &-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}\\ &-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}\end{aligned}$$

```
[Out] -(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(
c*d+I*e)/(1-I*c*x))/e+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e-I*b*
(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e-1/2*b^2*po
lylog(3,1-2/(1-I*c*x))/e+1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x
))/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4968}

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = -\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e} + \frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e} - \frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e}$$

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (I*b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 -
I*c*x)])/(2*e) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*e)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} = &-\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}\\&+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e}\\&-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}\\&-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}\end{aligned}$$

Mathematica [F]

$$\int\frac{(a+b\arctan(cx))^2}{d+ex}\,dx=\int\frac{(a+b\arctan(cx))^2}{d+ex}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x),x]
[Out] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 68.26 (sec) , antiderivative size = 1199, normalized size of antiderivative = 5.38

method	result	size
derivativedivides	Expression too large to display	1199
default	Expression too large to display	1199
parts	Expression too large to display	1202

```
[In] int((a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
[Out] 1/c*(a^2*c*ln(c*e*x+c*d)/e+b^2*c*(ln(c*e*x+c*d)/e*arctan(c*x)^2-2/e*(1/2*ar
ctan(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e
+c*d)-1/4*I*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^
2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*(csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x
^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+
1)))-csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c
*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I
*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I
*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I
```

```
*c*x)^2/(c^2*x^2+1)))+csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/
(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2)*arctan(c*x)^2-1/2*I*ar
ctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*polylog(3,-(1+I*c*x)^2/(c
^2*x^2+1))+1/2*I*c*d/(c*d-I*e)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1
+I*c*x)^2/(c^2*x^2+1))-1/2*c*d/(c*d-I*e)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+
I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/4*c*d/(c*d-I*e)*polylog(3,(I*e-c*d)/(c*d+I*
e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*e*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*
e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)-1/2*e*arctan(c*x)^2*ln(1-(I*e-c*d)/(c
*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)-1/4*e*polylog(3,(I*e-c*d)/(c*d+I
*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)))+2*a*b*c*(ln(c*e*x+c*d)/e*arctan(c*
x)-1/2*I*ln(c*e*x+c*d)*(-ln((I*e-c*e*x)/(c*d+I*e))+ln((I*e+c*e*x)/(I*e-c*d)
))/e+1/2*I*(dilog((I*e-c*e*x)/(c*d+I*e))-dilog((I*e+c*e*x)/(I*e-c*d)))/e))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{\left(a+b\operatorname{atan}\left(cx\right)\right)^2}{d+ex}\,dx$$

```
[In] integrate((a+b*atan(c*x))**2/(e*x+d),x)
[Out] Integral((a + b*atan(c*x))**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
[Out] a^2*log(e*x + d)/e + integrate(1/16*(12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2
 + 1)^2 + 32*a*b*arctan(c*x))/(e*x + d), x)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{d+e\,x}\,dx$$

```
[In] int((a + b*atan(c*x))^2/(d + e*x),x)
[Out] int((a + b*atan(c*x))^2/(d + e*x), x)
```

3.13 $\int \frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 341

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx = \frac{ic(a+b\arctan(cx))^2}{c^2d^2+e^2}+\frac{c^2d(a+b\arctan(cx))^2}{e(c^2d^2+e^2)} - \frac{(a+b\arctan(cx))^2}{e(d+ex)}-\frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2}+\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2+e^2} - \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2}$$

```
[Out] I*c*(a+b*arctan(c*x))^2/(c^2*d^2+e^2)+c^2*d*(a+b*arctan(c*x))^2/e/(c^2*d^2+
e^2)-(a+b*arctan(c*x))^2/e/(e*x+d)-2*b*c*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/
(c^2*d^2+e^2)+2*b*c*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/(c^2*d^2+e^2)+2*b*c*(
a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d^2+e^2)+I*b^2*c*
polylog(2,1-2/(1-I*c*x))/(c^2*d^2+e^2)+I*b^2*c*polylog(2,1-2/(1+I*c*x))/(c^
2*d^2+e^2)-I*b^2*c*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d^2+e^
2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4974, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964}

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx = \frac{ic(a+b\arctan(cx))^2}{c^2d^2+e^2}+\frac{c^2d(a+b\arctan(cx))^2}{e\left(c^2d^2+e^2\right)} - \frac{2bc\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c^2d^2+e^2} + \frac{2bc\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{c^2d^2+e^2} - \frac{(a+b\arctan(cx))^2}{e(d+ex)} + \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2d^2+e^2} - \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2}$$

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x)^2,x]
```

```
[Out] (I*c*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2) + (c^2*d*(a + b*ArcTan[c*x])^2)
/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])^2/(e*(d + e*x)) - (2*b*c*(a + b*
ArcTan[c*x])*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (2*b*c*(a + b*ArcTan[c*x
])*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2) + (2*b*c*(a + b*ArcTan[c*x])*Log[(2*
c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) + (I*b^2*c*PolyLog
[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (I*b^2*c*PolyLog[2, 1 - 2/(1 + I*
c*x)])/(c^2*d^2 + e^2) - (I*b^2*c*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*
e)*(1 - I*c*x))])/(c^2*d^2 + e^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\arctan(cx))^2}{e(d+ex)} + \frac{(2bc)\int \left(\frac{e^2(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)} + \frac{c^2(d-ex)(a+b\arctan(cx))}{(c^2d^2+e^2)(1+c^2x^2)}\right) dx}{e}$$

$$= -\frac{(a+b\arctan(cx))^2}{e(d+ex)} + \frac{(2bc^3)\int \frac{(d-ex)(a+b\arctan(cx))}{1+c^2x^2} dx}{e(c^2d^2+e^2)} + \frac{(2bce)\int \frac{a+b\arctan(cx)}{d+ex} dx}{c^2d^2+e^2}$$

$$= -\frac{(a+b\arctan(cx))^2}{e(d+ex)} - \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{(2b^2c^2)\int \frac{\log\left(\frac{2}{1-icx}\right)}{1+c^2x^2} dx}{c^2d^2+e^2} - \frac{(2b^2c^2)\int \frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{1+c^2x^2} dx}{c^2d^2+e^2} + \frac{(2bc^3)\int \left(\frac{d(a+b\arctan(cx))}{1+c^2x^2} - \frac{ex(a+b\arctan(cx))}{1+c^2x^2}\right) dx}{e(c^2d^2+e^2)}$$

$$= -\frac{(a+b\arctan(cx))^2}{e(d+ex)} - \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} - \frac{ib^2c\,\text{PolyLog}\left(2, 1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{(2ib^2c)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x} dx, x, \frac{1}{1-icx}\right)}{c^2d^2+e^2} - \frac{(2bc^3)\int \frac{x(a+b\arctan(cx))}{1+c^2x^2} dx}{c^2d^2+e^2} + \frac{(2bc^3d)\int \frac{a+b\arctan(cx)}{1+c^2x^2} dx}{e(c^2d^2+e^2)}$$

$$= \frac{ic(a+b\arctan(cx))^2}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^2}{e(c^2d^2+e^2)} - \frac{(a+b\arctan(cx))^2}{e(d+ex)} - \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{ib^2c\,\text{PolyLog}\left(2, 1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} - \frac{ib^2c\,\text{PolyLog}\left(2, 1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{(2bc^2)\int \frac{a+b\arctan(cx)}{i-cx} dx}{c^2d^2+e^2}$$

$$
\begin{aligned}
&= \frac{ic(a+b\arctan(cx))^2}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^2}{e\left(c^2d^2+e^2\right)} - \frac{(a+b\arctan(cx))^2}{e(d+ex)} \\
&\quad - \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2+e^2} \\
&\quad + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} \\
&\quad - \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} - \frac{\left(2b^2c^2\right)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2d^2+e^2} \\
&= \frac{ic(a+b\arctan(cx))^2}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^2}{e\left(c^2d^2+e^2\right)} - \frac{(a+b\arctan(cx))^2}{e(d+ex)} \\
&\quad - \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2+e^2} \\
&\quad + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} \\
&\quad - \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{\left(2ib^2c\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+icx}\right)}{c^2d^2+e^2} \\
&= \frac{ic(a+b\arctan(cx))^2}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^2}{e\left(c^2d^2+e^2\right)} - \frac{(a+b\arctan(cx))^2}{e(d+ex)} \\
&\quad - \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2+e^2} \\
&\quad + \frac{2bc(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} \\
&\quad + \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2+e^2} - \frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.88

$$
\begin{aligned}
&\int\frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx \\
&= -\frac{a^2}{e(d+ex)} + \frac{ab\left(-2(e-c^2dx)\arctan(cx)+c(d+ex)\left(2\log(c(d+ex))-\log\left(1+c^2x^2\right)\right)\right)}{\left(c^2d^2+e^2\right)(d+ex)} \\
&\quad + \frac{b^2\left(-\frac{e^{i\arctan\left(\frac{cd}{e}\right)}\arctan(cx)^2}{\sqrt{1+\frac{c^2d^2}{e^2}}\,e} + \frac{x\arctan(cx)^2}{d+ex} - \frac{cd\left(-i\left(\pi-2\arctan\left(\frac{cd}{e}\right)\right)\arctan(cx)-\pi\log\left(1+e^{-2i\arctan(cx)}\right)-2\left(\arctan\left(\frac{cd}{e}\right)+\text{arc}\right.\right.}{}\right.}{}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -(a^2/(e*(d + e*x))) + (a*b*(-2*(e - c^2*d*x)*ArcTan[c*x] + c*(d + e*x)*(2*
Log[c*(d + e*x)] - Log[1 + c^2*x^2])))/((c^2*d^2 + e^2)*(d + e*x)) + (b^2*(
-((E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2)/(Sqrt[1 + (c^2*d^2)/e^2]*e)) + (x*A
rcTan[c*x]^2)/(d + e*x) - (c*d*((-I)*(Pi - 2*ArcTan[(c*d)/e])*ArcTan[c*x] -
 Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] - 2*(ArcTan[(c*d)/e] + ArcTan[c*x])*Log
[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] - (Pi*Log[1 + c^2*x^2])/2 +
 2*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] + I*PolyLog[2, E
^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))]))/(c^2*d^2 + e^2)))/d
```

Maple [A] (verified)

Time = 21.77 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{-\frac{a^2c^2}{(cex+cd)e}+b^2c^2\left(-\frac{\arctan(cx)^2}{(cex+cd)e}+\frac{\frac{2\arctan(cx)e\ln(cex+cd)}{c^2d^2+e^2}-\frac{\arctan(cx)e\ln\left(c^2x^2+1\right)}{c^2d^2+e^2}+\frac{2\arctan(cx)^2cd}{2c^2d^2+2e^2}-\frac{2e^2\left(\frac{i\ln(cex+cd)\left(-\ln\right.}{}\right.}{}}{}\right.}{}$
default	$\frac{-\frac{a^2c^2}{(cex+cd)e}+b^2c^2\left(-\frac{\arctan(cx)^2}{(cex+cd)e}+\frac{\frac{2\arctan(cx)e\ln(cex+cd)}{c^2d^2+e^2}-\frac{\arctan(cx)e\ln\left(c^2x^2+1\right)}{c^2d^2+e^2}+\frac{2\arctan(cx)^2cd}{2c^2d^2+2e^2}-\frac{2e^2\left(\frac{i\ln(cex+cd)\left(-\ln\right.}{}\right.}{}}{}\right.}{}$
parts	$-\frac{a^2}{(ex+d)e}+\frac{b^2\left(-\frac{c^2\arctan(cx)^2}{(cex+cd)e}+\frac{2c^2\left(\frac{\arctan(cx)e\ln(cex+cd)}{c^2d^2+e^2}-\frac{\arctan(cx)e\ln\left(c^2x^2+1\right)}{2\left(c^2d^2+e^2\right)}+\frac{\arctan(cx)^2cd}{2c^2d^2+2e^2}-\frac{e^2\left(-\frac{i\ln(cex+cd)}{}\right.}{}\right.}{}\right.}{}$

```
[In] int((a+b*arctan(c*x))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a^2*c^2/(c*e*x+c*d)/e+b^2*c^2*(-1/(c*e*x+c*d)/e*arctan(c*x)^2+2/e*(ar
ctan(c*x)*e/(c^2*d^2+e^2)*ln(c*e*x+c*d)-1/2*arctan(c*x)/(c^2*d^2+e^2)*e*ln(
c^2*x^2+1)+1/2/(c^2*d^2+e^2)*d*c*arctan(c*x)^2-e^2/(c^2*d^2+e^2)*(1/2*I*ln(
```

```
c*e*x+c*d)*(-ln((I*e-c*e*x)/(c*d+I*e))+ln((I*e+c*e*x)/(I*e-c*d)))/e-1/2*I*(
dilog((I*e-c*e*x)/(c*d+I*e))-dilog((I*e+c*e*x)/(I*e-c*d)))/e)+1/2*e/(c^2*d^
2+e^2)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(
-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-dilog(1/2*I
*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2))))+2*a*b*c^2*(-1/(c*
e*x+c*d)/e*arctan(c*x)+1/e*(e/(c^2*d^2+e^2)*ln(c*e*x+c*d)+1/(c^2*d^2+e^2)*(
-1/2*e*ln(c^2*x^2+1)+d*c*arctan(c*x)))))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(ex+d\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^2 + 2*d*e*x +
 d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx = \int \frac{\left(a+b\operatorname{atan}\left(cx\right)\right)^2}{\left(d+ex\right)^2}\,dx$$

```
[In] integrate((a+b*atan(c*x))**2/(e*x+d)**2,x)
[Out] Integral((a + b*atan(c*x))**2/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(ex+d\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
[Out] ((2*c*d*arctan(c*x)/(c^2*d^2*e + e^3) - log(c^2*x^2 + 1)/(c^2*d^2 + e^2) +
2*log(e*x + d)/(c^2*d^2 + e^2))*c - 2*arctan(c*x)/(e^2*x + d*e))*a*b - 1/16
*(4*arctan(c*x)^2 - 16*(e^2*x + d*e)*integrate(1/16*(12*(c^2*e*x^2 + e)*arc
tan(c*x)^2 + (c^2*e*x^2 + e)*log(c^2*x^2 + 1)^2 + 8*(c*e*x + c*d)*arctan(c*
x) - 4*(c^2*e*x^2 + c^2*d*x)*log(c^2*x^2 + 1))/(c^2*e^3*x^4 + 2*c^2*d*e^2*x
^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2), x) - log(c^2*x^2 + 1)^2)*b
^2/(e^2*x + d*e) - a^2/(e^2*x + d*e)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(ex+d\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{\left(d+e\,x\right)^2}\,dx$$

```
[In] int((a + b*atan(c*x))^2/(d + e*x)^2,x)
[Out] int((a + b*atan(c*x))^2/(d + e*x)^2, x)
```

3.14 $\int \frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 496

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx = \frac{b^2c^3d\arctan(cx)}{(c^2d^2+e^2)^2} - \frac{bc(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^2}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^2}{2e(c^2d^2+e^2)^2} - \frac{(a+b\arctan(cx))^2}{2e(d+ex)^2} - \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{b^2c^2e\log(d+ex)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} - \frac{b^2c^2e\log(1+c^2x^2)}{2(c^2d^2+e^2)^2} + \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} - \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$

```
[Out] b^2*c^3*d*arctan(c*x)/(c^2*d^2+e^2)^2-b*c*(a+b*arctan(c*x))/(c^2*d^2+e^2)/(
e*x+d)+I*c^3*d*(a+b*arctan(c*x))^2/(c^2*d^2+e^2)^2+1/2*c^2*(c*d-e)*(c*d+e)*
(a+b*arctan(c*x))^2/e/(c^2*d^2+e^2)^2-1/2*(a+b*arctan(c*x))^2/e/(e*x+d)^2-2
```

```
*b*c^3*d*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/(c^2*d^2+e^2)^2+2*b*c^3*d*(a+b*a
rctan(c*x))*ln(2/(1+I*c*x))/(c^2*d^2+e^2)^2+b^2*c^2*e*ln(e*x+d)/(c^2*d^2+e^
2)^2+2*b*c^3*d*(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d
^2+e^2)^2-1/2*b^2*c^2*e*ln(c^2*x^2+1)/(c^2*d^2+e^2)^2+I*b^2*c^3*d*polylog(2
,1-2/(1-I*c*x))/(c^2*d^2+e^2)^2+I*b^2*c^3*d*polylog(2,1-2/(1+I*c*x))/(c^2*d
^2+e^2)^2-I*b^2*c^3*d*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d^2
+e^2)^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4974, 4972, 720, 31, 649, 209, 266, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964}

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx = \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^2}{2e\left(c^2d^2+e^2\right)^2} - \frac{bc(a+b\arctan(cx))}{\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^2}{\left(c^2d^2+e^2\right)^2} - \frac{2bc^3d\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{\left(c^2d^2+e^2\right)^2} + \frac{2bc^3d\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{\left(c^2d^2+e^2\right)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{\left(c^2d^2+e^2\right)^2} - \frac{(a+b\arctan(cx))^2}{2e(d+ex)^2} + \frac{b^2c^3d\arctan(cx)}{\left(c^2d^2+e^2\right)^2} - \frac{b^2c^2e\log\left(c^2x^2+1\right)}{2\left(c^2d^2+e^2\right)^2} + \frac{b^2c^2e\log(d+ex)}{\left(c^2d^2+e^2\right)^2} + \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} + \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{\left(c^2d^2+e^2\right)^2} - \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2}$$

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x)^3,x]
```

```
[Out] (b^2*c^3*d*ArcTan[c*x])/(c^2*d^2 + e^2)^2 - (b*c*(a + b*ArcTan[c*x]))/((c^2
*d^2 + e^2)*(d + e*x)) + (I*c^3*d*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2)^2
+ (c^2*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2)/(2*e*(c^2*d^2 + e^2)^2) -
 (a + b*ArcTan[c*x])^2/(2*e*(d + e*x)^2) - (2*b*c^3*d*(a + b*ArcTan[c*x])*L
og[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (2*b*c^3*d*(a + b*ArcTan[c*x])*Log[2
/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (b^2*c^2*e*Log[d + e*x])/(c^2*d^2 + e^2)
^2 + (2*b*c^3*d*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I
*c*x))])/(c^2*d^2 + e^2)^2 - (b^2*c^2*e*Log[1 + c^2*x^2])/(2*(c^2*d^2 + e^2
```

```
)^2) + (I*b^2*c^3*d*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (I*b
^2*c^3*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 - (I*b^2*c^3*d*Po
lyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))^2}{2e(d+ex)^2} \\
&\quad + \frac{(bc)\int\left(\frac{e^2(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)^2}+\frac{2c^2de^2(a+b\arctan(cx))}{(c^2d^2+e^2)^2(d+ex)}+\frac{(c^4d^2-c^2e^2-2c^4dex)(a+b\arctan(cx))}{(c^2d^2+e^2)^2(1+c^2x^2)}\right)dx}{e} \\
&= -\frac{(a+b\arctan(cx))^2}{2e(d+ex)^2}+\frac{(bc)\int\frac{(c^4d^2-c^2e^2-2c^4dex)(a+b\arctan(cx))}{1+c^2x^2}\,dx}{e\left(c^2d^2+e^2\right)^2} \\
&\quad + \frac{(2bc^3de)\int\frac{a+b\arctan(cx)}{d+ex}\,dx}{\left(c^2d^2+e^2\right)^2}+\frac{(bce)\int\frac{a+b\arctan(cx)}{(d+ex)^2}\,dx}{c^2d^2+e^2} \\
&= -\frac{bc(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)}-\frac{(a+b\arctan(cx))^2}{2e(d+ex)^2}-\frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} \\
&\quad + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2} \\
&\quad + \frac{(2b^2c^4d)\int\frac{\log\left(\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx}{\left(c^2d^2+e^2\right)^2}-\frac{(2b^2c^4d)\int\frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{1+c^2x^2}\,dx}{\left(c^2d^2+e^2\right)^2} \\
&\quad + \frac{(bc)\int\left(\frac{c^4d^2\left(1-\frac{e^2}{c^2d^2}\right)(a+b\arctan(cx))}{1+c^2x^2}-\frac{2c^4dex(a+b\arctan(cx))}{1+c^2x^2}\right)dx}{e\left(c^2d^2+e^2\right)^2} \\
&\quad + \frac{(b^2c^2)\int\frac{1}{(d+ex)(1+c^2x^2)}\,dx}{c^2d^2+e^2}
\end{aligned}
$$

$$= -\frac{bc(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\arctan(cx))^2}{2e(d+ex)^2} - \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} - \frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} + \frac{(b^2c^2)\int\frac{c^2d-c^2ex}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2} + \frac{(2ib^2c^3d)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-icx}\right)}{(c^2d^2+e^2)^2} - \frac{(2bc^5d)\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2} + \frac{(b^2c^2e^2)\int\frac{1}{d+ex}\,dx}{(c^2d^2+e^2)^2} + \frac{(bc^3(cd-e)(cd+e))\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{e(c^2d^2+e^2)^2}$$

$$= -\frac{bc(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^2}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^2}{2e(c^2d^2+e^2)^2} - \frac{(a+b\arctan(cx))^2}{2e(d+ex)^2} - \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{b^2c^2e\log(d+ex)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} + \frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} - \frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} + \frac{(2bc^4d)\int\frac{a+b\arctan(cx)}{i-cx}\,dx}{(c^2d^2+e^2)^2} + \frac{(b^2c^4d)\int\frac{1}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2} - \frac{(b^2c^4e)\int\frac{x}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2}$$

$$= \frac{b^2c^3d\arctan(cx)}{(c^2d^2+e^2)^2} - \frac{bc(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^2}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^2}{2e(c^2d^2+e^2)^2} - \frac{(a+b\arctan(cx))^2}{2e(d+ex)^2} - \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{b^2c^2e\log(d+ex)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} - \frac{b^2c^2e\log(1+c^2x^2)}{2(c^2d^2+e^2)^2} + \frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} - \frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} - \frac{(2b^2c^4d)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2}$$

$$= \frac{b^2c^3d\arctan(cx)}{(c^2d^2+e^2)^2} - \frac{bc(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^2}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^2}{2e(c^2d^2+e^2)^2} - \frac{(a+b\arctan(cx))^2}{2e(d+ex)^2} - \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{b^2c^2e\log(d+ex)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} - \frac{b^2c^2e\log(1+c^2x^2)}{2(c^2d^2+e^2)^2} + \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} - \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} + \frac{(2ib^2c^3d)\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+icx}\right)}{(c^2d^2+e^2)^2}$$

$$= \frac{b^2c^3d\arctan(cx)}{(c^2d^2+e^2)^2} - \frac{bc(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^2}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^2}{2e(c^2d^2+e^2)^2} - \frac{(a+b\arctan(cx))^2}{2e(d+ex)^2} - \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{b^2c^2e\log(d+ex)}{(c^2d^2+e^2)^2} + \frac{2bc^3d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} - \frac{b^2c^2e\log(1+c^2x^2)}{2(c^2d^2+e^2)^2} + \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} - \frac{ib^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$

Mathematica [A] (verified)

Time = 6.54 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.97

$$\int\frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx = -\frac{a^2}{2e(d+ex)^2} + \frac{ab((-e^3+c^4d^2x(2d+ex)-c^2e(3d^2+2dex+e^2x^2))\arctan(cx)+c(d+ex)(-c^2d^2-e^2+2c^2d(d+e}{(c^2d^2+e^2)^2(d+ex)^2} + \frac{b^2c^2\left(-\frac{2e^{i\arctan\left(\frac{cd}{e}\right)}\arctan(cx)^2}{\sqrt{1+\frac{c^2d^2}{e^2}}e} - \frac{e(1+c^2x^2)\arctan(cx)^2}{c^2(d+ex)^2} + \frac{2x\arctan(cx)(e+cd\arctan(cx))}{cd(d+ex)} + \frac{-2e^2\arctan(cx)+2cde\log\left(\frac{c(d+ex}{\sqrt{1+c^2}}\right.}{c^3d^3+cde^2}\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -1/2*a^2/(e*(d + e*x)^2) + (a*b*((-e^3 + c^4*d^2*x*(2*d + e*x) - c^2*e*(3*d
^2 + 2*d*e*x + e^2*x^2))*ArcTan[c*x] + c*(d + e*x)*(-(c^2*d^2) - e^2 + 2*c^
2*d*(d + e*x)*Log[c*(d + e*x)] - c^2*d*(d + e*x)*Log[1 + c^2*x^2])))/((c^2*
d^2 + e^2)^2*(d + e*x)^2) + (b^2*c^2*((-2*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]
^2)/(Sqrt[1 + (c^2*d^2)/e^2]*e) - (e*(1 + c^2*x^2)*ArcTan[c*x]^2)/(c^2*(d +
 e*x)^2) + (2*x*ArcTan[c*x]*(e + c*d*ArcTan[c*x]))/(c*d*(d + e*x)) + (-2*e^
2*ArcTan[c*x] + 2*c*d*e*Log[(c*(d + e*x))/Sqrt[1 + c^2*x^2]])/(c^3*d^3 + c*
d*e^2) - (2*c*d*((-I)*(Pi - 2*ArcTan[(c*d)/e])*ArcTan[c*x] - Pi*Log[1 + E^(
(-2*I)*ArcTan[c*x])] - 2*(ArcTan[(c*d)/e] + ArcTan[c*x])*Log[1 - E^((2*I)*(
ArcTan[(c*d)/e] + ArcTan[c*x]))] - (Pi*Log[1 + c^2*x^2])/2 + 2*ArcTan[(c*d)
/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] + I*PolyLog[2, E^((2*I)*(ArcTan
[(c*d)/e] + ArcTan[c*x]))]))/(c^2*d^2 + e^2)))/(2*(c^2*d^2 + e^2))
```

Maple [A] (verified)

Time = 28.13 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.47

method	result
derivativedivides	$-\frac{a^2c^3}{2(cex+cd)^2e}+b^2c^3\left(-\frac{\arctan(cx)^2}{2(cex+cd)^2e}+\frac{-\frac{\arctan(cx)e}{(c^2d^2+e^2)(cex+cd)}+\frac{2\arctan(cx)ecd\ln(cex+cd)}{(c^2d^2+e^2)^2}+\frac{\arctan(cx)^2c^2d^2}{(c^2d^2+e^2)^2}-\frac{\arctan(cx)\ln(c^2}{(c^2d^2+e^2}\right.$
default	$-\frac{a^2c^3}{2(cex+cd)^2e}+b^2c^3\left(-\frac{\arctan(cx)^2}{2(cex+cd)^2e}+\frac{-\frac{\arctan(cx)e}{(c^2d^2+e^2)(cex+cd)}+\frac{2\arctan(cx)ecd\ln(cex+cd)}{(c^2d^2+e^2)^2}+\frac{\arctan(cx)^2c^2d^2}{(c^2d^2+e^2)^2}-\frac{\arctan(cx)\ln(c^2}{(c^2d^2+e^2}\right.$
parts	$-\frac{a^2}{2(ex+d)^2e}+\frac{b^2\left(-\frac{c^3\arctan(cx)^2}{2(cex+cd)^2e}+\frac{c^3\left(-\frac{\arctan(cx)e}{(c^2d^2+e^2)(cex+cd)}+\frac{2\arctan(cx)ecd\ln(cex+cd)}{(c^2d^2+e^2)^2}+\frac{\arctan(cx)^2c^2d^2}{(c^2d^2+e^2)^2}-\frac{\arctan(cx)}{(c^2}\right.}{}\right.}{}$

```
[In] int((a+b*arctan(c*x))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a^2*c^3/(c*e*x+c*d)^2/e+b^2*c^3*(-1/2/(c*e*x+c*d)^2/e*arctan(c*x)
^2+1/e*(-arctan(c*x)*e/(c^2*d^2+e^2)/(c*e*x+c*d)+2*arctan(c*x)*e*c*d/(c^2*d
^2+e^2)^2*ln(c*e*x+c*d)+1/(c^2*d^2+e^2)^2*arctan(c*x)^2*c^2*d^2-arctan(c*x)
/(c^2*d^2+e^2)^2*ln(c^2*x^2+1)*c*d*e-1/(c^2*d^2+e^2)^2*arctan(c*x)^2*e^2-1/
2/(c^2*d^2+e^2)^2*(c^2*d^2-e^2)*arctan(c*x)^2+e^2/(c^2*d^2+e^2)^2*ln(c*e*x+
c*d)-1/2*e^2/(c^2*d^2+e^2)^2*ln(c^2*x^2+1)+e/(c^2*d^2+e^2)^2*d*c*arctan(c*x
)+e*c*d/(c^2*d^2+e^2)^2*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+
I))-ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)+1/2*I*(ln(c*x+I)*ln(c^2*x
^2+1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2))-2*
e^2*c*d/(c^2*d^2+e^2)^2*(1/2*I*ln(c*e*x+c*d)*(-ln((I*e-c*e*x)/(c*d+I*e))+ln
((I*e+c*e*x)/(I*e-c*d)))/e-1/2*I*(dilog((I*e-c*e*x)/(c*d+I*e))-dilog((I*e+c
*e*x)/(I*e-c*d)))/e)))+2*a*b*c^3*(-1/2/(c*e*x+c*d)^2/e*arctan(c*x)+1/2/e*(-
e/(c^2*d^2+e^2)/(c*e*x+c*d)+2*e*c*d/(c^2*d^2+e^2)^2*ln(c*e*x+c*d)+1/(c^2*d^
2+e^2)^2*(-c*d*e*ln(c^2*x^2+1)+(c^2*d^2-e^2)*arctan(c*x)))))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^3*x^3 + 3*d*e^2*x
^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(c*x))**2/(e*x+d)**3,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex)^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(d+e\,x)^3}\,dx$$

```
[In] int((a + b*atan(c*x))^2/(d + e*x)^3,x)
[Out] int((a + b*atan(c*x))^2/(d + e*x)^3, x)
```

3.15 $\int (d+ex)^3(a+b\arctan(cx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 652

$$\begin{aligned}
&\int (d+ex)^3(a+b\arctan(cx))^3\,dx\\
&=\frac{3ab^2de^2x}{c^2}-\frac{b^3e^3x}{4c^3}+\frac{b^3e^3\arctan(cx)}{4c^4}+\frac{3b^3de^2x\arctan(cx)}{c^2}\\
&+\frac{b^2e^3x^2(a+b\arctan(cx))}{4c^2}-\frac{3bde^2(a+b\arctan(cx))^2}{2c^3}\\
&+\frac{ibe^3(a+b\arctan(cx))^2}{4c^4}-\frac{3ibe(6c^2d^2-e^2)\,(a+b\arctan(cx))^2}{4c^4}\\
&-\frac{3be(6c^2d^2-e^2)\,x(a+b\arctan(cx))^2}{4c^3}-\frac{3bde^2x^2(a+b\arctan(cx))^2}{2c}\\
&-\frac{be^3x^3(a+b\arctan(cx))^2}{4c}+\frac{id(cd-e)(cd+e)(a+b\arctan(cx))^3}{c^3}\\
&-\frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^3}{4c^4e}+\frac{(d+ex)^4(a+b\arctan(cx))^3}{4e}\\
&+\frac{b^2e^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4}-\frac{3b^2e(6c^2d^2-e^2)\,(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4}\\
&+\frac{3bd(cd-e)(cd+e)(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3}-\frac{3b^3de^2\log\left(1+c^2x^2\right)}{2c^3}\\
&+\frac{ib^3e^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{4c^4}-\frac{3ib^3e(6c^2d^2-e^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{4c^4}\\
&+\frac{3ib^2d(cd-e)(cd+e)(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3}\\
&+\frac{3b^3d(cd-e)(cd+e)\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}
\end{aligned}$$

```
[Out] 3*a*b^2*d*e^2*x/c^2-1/4*b^3*e^3*x/c^3+1/4*b^3*e^3*arctan(c*x)/c^4+3*b^3*d*e
^2*x*arctan(c*x)/c^2+1/4*b^2*e^3*x^2*(a+b*arctan(c*x))/c^2-3/2*b*d*e^2*(a+b
```

```
*arctan(c*x))^2/c^3-3/4*I*b*e*(6*c^2*d^2-e^2)*(a+b*arctan(c*x))^2/c^4+1/4*I
*b*e^3*(a+b*arctan(c*x))^2/c^4-3/4*b*e*(6*c^2*d^2-e^2)*x*(a+b*arctan(c*x))^
2/c^3-3/2*b*d*e^2*x^2*(a+b*arctan(c*x))^2/c-1/4*b*e^3*x^3*(a+b*arctan(c*x))
^2/c-3/4*I*b^3*e*(6*c^2*d^2-e^2)*polylog(2,1-2/(1+I*c*x))/c^4-1/4*(c^4*d^4-
6*c^2*d^2*e^2+e^4)*(a+b*arctan(c*x))^3/c^4/e+1/4*(e*x+d)^4*(a+b*arctan(c*x)
)^3/e+1/2*b^2*e^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4-3/2*b^2*e*(6*c^2*d^
2-e^2)*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4+3*b*d*(c*d-e)*(c*d+e)*(a+b*arc
tan(c*x))^2*ln(2/(1+I*c*x))/c^3-3/2*b^3*d*e^2*ln(c^2*x^2+1)/c^3+3*I*b^2*d*(
c*d-e)*(c*d+e)*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3+I*d*(c*d-e)*(
c*d+e)*(a+b*arctan(c*x))^3/c^3+1/4*I*b^3*e^3*polylog(2,1-2/(1+I*c*x))/c^4+3
/2*b^3*d*(c*d-e)*(c*d+e)*polylog(3,1-2/(1+I*c*x))/c^3
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4974, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 327, 209, 5104, 5114, 6745}

$$\int (d+ex)^3(a+b\arctan(cx))^3\,dx$$
$$= \frac{b^2e^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{2c^4}$$
$$+ \frac{3ib^2d(cd-e)(cd+e)\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^3}$$
$$+ \frac{b^2e^3x^2(a+b\arctan(cx))}{4c^2} - \frac{3b^2e(6c^2d^2-e^2)\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{2c^4}$$
$$+ \frac{ibe^3(a+b\arctan(cx))^2}{4c^4} - \frac{3bde^2(a+b\arctan(cx))^2}{2c^3}$$
$$+ \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^3}{c^3}$$
$$+ \frac{3bd(cd-e)(cd+e)\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^3}$$
$$- \frac{3ibe(6c^2d^2-e^2)(a+b\arctan(cx))^2}{4c^4} - \frac{(c^4d^4-6c^2d^2e^2+e^4)(a+b\arctan(cx))^3}{4c^4e}$$
$$- \frac{3bex(6c^2d^2-e^2)(a+b\arctan(cx))^2}{4c^3} - \frac{3bde^2x^2(a+b\arctan(cx))^2}{2c}$$
$$+ \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} - \frac{be^3x^3(a+b\arctan(cx))^2}{4c} + \frac{3ab^2de^2x}{c^2}$$
$$+ \frac{b^3e^3\arctan(cx)}{4c^4} + \frac{3b^3de^2x\arctan(cx)}{c^2} + \frac{ib^3e^3\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{4c^4}$$
$$+ \frac{3b^3d(cd-e)(cd+e)\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^3} - \frac{b^3e^3x}{4c^3}$$
$$- \frac{3ib^3e(6c^2d^2-e^2)\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{4c^4} - \frac{3b^3de^2\log(c^2x^2+1)}{2c^3}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (3*a*b^2*d*e^2*x)/c^2 - (b^3*e^3*x)/(4*c^3) + (b^3*e^3*ArcTan[c*x])/(4*c^4)
 + (3*b^3*d*e^2*x*ArcTan[c*x])/c^2 + (b^2*e^3*x^2*(a + b*ArcTan[c*x]))/(4*c
^2) - (3*b*d*e^2*(a + b*ArcTan[c*x])^2)/(2*c^3) + ((I/4)*b*e^3*(a + b*ArcTa
n[c*x])^2)/c^4 - (((3*I)/4)*b*e*(6*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])^2)/c^
4 - (3*b*e*(6*c^2*d^2 - e^2)*x*(a + b*ArcTan[c*x])^2)/(4*c^3) - (3*b*d*e^2*
x^2*(a + b*ArcTan[c*x])^2)/(2*c) - (b*e^3*x^3*(a + b*ArcTan[c*x])^2)/(4*c)
+ (I*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^3)/c^3 - ((c^4*d^4 - 6*c^2*d
^2*e^2 + e^4)*(a + b*ArcTan[c*x])^3)/(4*c^4*e) + ((d + e*x)^4*(a + b*ArcTan
[c*x])^3)/(4*e) + (b^2*e^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(2*c^4)
- (3*b^2*e*(6*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(2*c^4
) + (3*b*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c^
3 - (3*b^3*d*e^2*Log[1 + c^2*x^2])/(2*c^3) + ((I/4)*b^3*e^3*PolyLog[2, 1 -
2/(1 + I*c*x)])/c^4 - (((3*I)/4)*b^3*e*(6*c^2*d^2 - e^2)*PolyLog[2, 1 - 2/(
```

```
1 + I*c*x)])/c^4 + ((3*I)*b^2*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (3*b^3*d*(c*d - e)*(c*d + e)*PolyLog[3, 1
 - 2/(1 + I*c*x)])/(2*c^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} - \frac{(3bc)\int\left(\frac{e^2(6c^2d^2-e^2)(a+b\arctan(cx))^2}{c^4}+\frac{4de^3x(a+b\arctan(cx))^2}{c^2}+\frac{e^4x^2(a+b\arctan(cx))^2}{c^2}+\frac{(c^4d^4-6c^2d^2e^2+e^4+4c^2d(cd-e)e(c}{c^4(1+c^2x^2}\right.}{4e}$$

$$= \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} - \frac{(3b)\int\frac{(c^4d^4-6c^2d^2e^2+e^4+4c^2d(cd-e)e(cd+e)x)(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{4c^3e} - \frac{(3bde^2)\int x(a+b\arctan(cx))^2\,dx}{c} - \frac{(3be^3)\int x^2(a+b\arctan(cx))^2\,dx}{4c} - \frac{(3be(6c^2d^2-e^2))\int(a+b\arctan(cx))^2\,dx}{4c^3}$$

$$= -\frac{3be(6c^2d^2-e^2)\,x(a+b\arctan(cx))^2}{4c^3} - \frac{3bde^2x^2(a+b\arctan(cx))^2}{2c} - \frac{be^3x^3(a+b\arctan(cx))^2}{4c} + \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} - \frac{(3b)\int\left(\frac{c^4d^4\left(1+\frac{-6c^2d^2e^2+e^4}{c^4d^4}\right)(a+b\arctan(cx))^2}{1+c^2x^2}+\frac{4c^2d(cd-e)e(cd+e)x(a+b\arctan(cx))^2}{1+c^2x^2}\right)dx}{4c^3e} + (3b^2de^2)\int\frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx + \frac{1}{2}(b^2e^3)\int\frac{x^3(a+b\arctan(cx))}{1+c^2x^2}\,dx + \frac{(3b^2e(6c^2d^2-e^2))\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{2c^2}$$

$$= -\frac{3ibe(6c^2d^2-e^2)\,(a+b\arctan(cx))^2}{4c^4} - \frac{3be(6c^2d^2-e^2)\,x(a+b\arctan(cx))^2}{4c^3} - \frac{3bde^2x^2(a+b\arctan(cx))^2}{2c} - \frac{be^3x^3(a+b\arctan(cx))^2}{4c} + \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} + \frac{(3b^2de^2)\int(a+b\arctan(cx))\,dx}{c^2} - \frac{(3b^2de^2)\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{c^2} + \frac{(b^2e^3)\int x(a+b\arctan(cx))\,dx}{2c^2} - \frac{(b^2e^3)\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{2c^2} - \frac{(3bd(cd-e)(cd+e))\int\frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{c} - \frac{(3b^2e(6c^2d^2-e^2))\int\frac{a+b\arctan(cx)}{i-cx}\,dx}{2c^3} - \frac{(3b(c^4d^4-6c^2d^2e^2+e^4))\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{4c^3e}$$

$$= \frac{3ab^2de^2x}{c^2} + \frac{b^2e^3x^2(a+b\arctan(cx))}{4c^2} - \frac{3bde^2(a+b\arctan(cx))^2}{2c^3} + \frac{ibe^3(a+b\arctan(cx))^2}{4c^4} - \frac{3ibe(6c^2d^2-e^2)\,(a+b\arctan(cx))^2}{4c^4} - \frac{3be(6c^2d^2-e^2)\,x(a+b\arctan(cx))^2}{4c^3} - \frac{3bde^2x^2(a+b\arctan(cx))^2}{2c} - \frac{be^3x^3(a+b\arctan(cx))^2}{4c} + \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^3}{c^3} - \frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^3}{4c^4e} + \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} - \frac{3b^2e(6c^2d^2-e^2)\,(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4} + \frac{(3b^3de^2)\int\arctan(cx)\,dx}{c^2} + \frac{(b^2e^3)\int\frac{a+b\arctan(cx)}{i-cx}\,dx}{2c^3} - \frac{(b^3e^3)\int\frac{x^2}{1+c^2x^2}\,dx}{4c} + \frac{(3bd(cd-e)(cd+e))\int\frac{(a+b\arctan(cx))^2}{i-cx}\,dx}{c^2} + \frac{(3b^3e(6c^2d^2-e^2))\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{2c^3}$$

$$
\begin{aligned}
&= \frac{3ab^2de^2x}{c^2} - \frac{b^3e^3x}{4c^3} + \frac{3b^3de^2x\arctan(cx)}{c^2} + \frac{b^2e^3x^2(a+b\arctan(cx))}{4c^2} \\
&\quad - \frac{3bde^2(a+b\arctan(cx))^2}{2c^3} + \frac{ibe^3(a+b\arctan(cx))^2}{4c^4} \\
&\quad - \frac{3ibe(6c^2d^2-e^2)\,(a+b\arctan(cx))^2}{4c^4} - \frac{3be(6c^2d^2-e^2)\,x(a+b\arctan(cx))^2}{4c^3} \\
&\quad - \frac{3bde^2x^2(a+b\arctan(cx))^2}{2c} - \frac{be^3x^3(a+b\arctan(cx))^2}{4c} \\
&\quad + \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^3}{c^3} \\
&\quad - \frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^3}{4c^4e} \\
&\quad + \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} + \frac{b^2e^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4} \\
&\quad - \frac{3b^2e(6c^2d^2-e^2)\,(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4} \\
&\quad + \frac{3bd(cd-e)(cd+e)(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} \\
&\quad - \frac{(3b^3de^2)\int\frac{x}{1+c^2x^2}\,dx}{c} + \frac{(b^3e^3)\int\frac{1}{1+c^2x^2}\,dx}{4c^3} - \frac{(b^3e^3)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{2c^3} \\
&\quad - \frac{(6b^2d(cd-e)(cd+e))\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2} \\
&\quad - \frac{(3ib^3e(6c^2d^2-e^2))\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{2c^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3ab^2de^2x}{c^2} - \frac{b^3e^3x}{4c^3} + \frac{b^3e^3\arctan(cx)}{4c^4} + \frac{3b^3de^2x\arctan(cx)}{c^2} \\
&+ \frac{b^2e^3x^2(a+b\arctan(cx))}{4c^2} - \frac{3bde^2(a+b\arctan(cx))^2}{2c^3} \\
&+ \frac{ibe^3(a+b\arctan(cx))^2}{4c^4} - \frac{3ibe(6c^2d^2-e^2)\,(a+b\arctan(cx))^2}{4c^4} \\
&- \frac{3be(6c^2d^2-e^2)\,x(a+b\arctan(cx))^2}{4c^3} - \frac{3bde^2x^2(a+b\arctan(cx))^2}{2c} \\
&- \frac{be^3x^3(a+b\arctan(cx))^2}{4c} + \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^3}{c^3} \\
&- \frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^3}{4c^4e} \\
&+ \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} + \frac{b^2e^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4} \\
&- \frac{3b^2e(6c^2d^2-e^2)\,(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4} \\
&+ \frac{3bd(cd-e)(cd+e)(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} \\
&- \frac{3b^3de^2\log(1+c^2x^2)}{2c^3} - \frac{3ib^3e(6c^2d^2-e^2)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{4c^4} \\
&+ \frac{3ib^2d(cd-e)(cd+e)(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} \\
&+ \frac{(ib^3e^3)\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{2c^4} \\
&- \frac{(3ib^3d(cd-e)(cd+e))\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3ab^2de^2x}{c^2} - \frac{b^3e^3x}{4c^3} + \frac{b^3e^3\arctan(cx)}{4c^4} + \frac{3b^3de^2x\arctan(cx)}{c^2} \\
&+ \frac{b^2e^3x^2(a+b\arctan(cx))}{4c^2} - \frac{3bde^2(a+b\arctan(cx))^2}{2c^3} \\
&+ \frac{ibe^3(a+b\arctan(cx))^2}{4c^4} - \frac{3ibe(6c^2d^2-e^2)\,(a+b\arctan(cx))^2}{4c^4} \\
&- \frac{3be(6c^2d^2-e^2)\,x(a+b\arctan(cx))^2}{4c^3} - \frac{3bde^2x^2(a+b\arctan(cx))^2}{2c} \\
&- \frac{be^3x^3(a+b\arctan(cx))^2}{4c} + \frac{id(cd-e)(cd+e)(a+b\arctan(cx))^3}{c^3} \\
&- \frac{(c^4d^4-6c^2d^2e^2+e^4)\,(a+b\arctan(cx))^3}{4c^4e} \\
&+ \frac{(d+ex)^4(a+b\arctan(cx))^3}{4e} + \frac{b^2e^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4} \\
&- \frac{3b^2e(6c^2d^2-e^2)\,(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{2c^4} \\
&+ \frac{3bd(cd-e)(cd+e)(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{3b^3de^2\log\left(1+c^2x^2\right)}{2c^3} \\
&+ \frac{ib^3e^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{4c^4} - \frac{3ib^3e(6c^2d^2-e^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{4c^4} \\
&+ \frac{3ib^2d(cd-e)(cd+e)(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} \\
&+ \frac{3b^3d(cd-e)(cd+e)\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.31

$$
\int (d+ex)^3(a+b\arctan(cx))^3\,dx
= \frac{a^2c(4ac^3d^3+3be(-6c^2d^2+e^2))\,x+6a^2c^3de(acd-be)x^2+a^2c^3e^2(4acd-be)x^3+a^3c^4e^3x^4+3a^2b(6c^2d^2e
$$

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a^2*c*(4*a*c^3*d^3 + 3*b*e*(-6*c^2*d^2 + e^2))*x + 6*a^2*c^3*d*e*(a*c*d -
b*e)*x^2 + a^2*c^3*e^2*(4*a*c*d - b*e)*x^3 + a^3*c^4*e^3*x^4 + 3*a^2*b*(6*c
^2*d^2*e - e^3)*ArcTan[c*x] + 3*a^2*b*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^
2 + e^3*x^3)*ArcTan[c*x] + a*b^2*e^3*(1 + c^2*x^2 + (6*c*x - 2*c^3*x^3)*Arc
Tan[c*x] + 3*(-1 + c^4*x^4)*ArcTan[c*x]^2 - 4*Log[1 + c^2*x^2]) - 6*a^2*b*c
*d*(c^2*d^2 - e^2)*Log[1 + c^2*x^2] + 18*a*b^2*c^2*d^2*e*(-2*c*x*ArcTan[c*x
] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]) + 12*a*b^2*c^3*d^3*(Arc
Tan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) - I*Po
```

```
lyLog[2, -E^((2*I)*ArcTan[c*x])]) + 12*a*b^2*c*d*e^2*(c*x + (I + c^3*x^3)*A
rcTan[c*x]^2 - ArcTan[c*x]*(1 + c^2*x^2 + 2*Log[1 + E^((2*I)*ArcTan[c*x])])
 + I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + 6*b^3*c^2*d^2*e*(ArcTan[c*x]*((3
*I - 3*c*x)*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 6*Log[1 + E^((2*I)*
ArcTan[c*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + b^3*e^3*(-(c*x
) - (4*I - 3*c*x + c^3*x^3)*ArcTan[c*x]^2 + (-1 + c^4*x^4)*ArcTan[c*x]^3 +
ArcTan[c*x]*(1 + c^2*x^2 + 8*Log[1 + E^((2*I)*ArcTan[c*x])]) - (4*I)*PolyLo
g[2, -E^((2*I)*ArcTan[c*x])]) + 2*b^3*c*d*e^2*(6*c*x*ArcTan[c*x] - 3*ArcTan
[c*x]^2 - 3*c^2*x^2*ArcTan[c*x]^2 + (2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[
c*x]^3 - 6*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2
] + (6*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^
((2*I)*ArcTan[c*x])]) + 2*b^3*c^3*d^3*(2*ArcTan[c*x]^2*((-I + c*x)*ArcTan[c
*x] + 3*Log[1 + E^((2*I)*ArcTan[c*x])]) - (6*I)*ArcTan[c*x]*PolyLog[2, -E^(
(2*I)*ArcTan[c*x])] + 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(4*c^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 93.44 (sec) , antiderivative size = 3122, normalized size of antiderivative = 4.79

method	result	size
parts	Expression too large to display	3122
derivativedivides	Expression too large to display	3153
default	Expression too large to display	3153

```
[In] int((e*x+d)^3*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^3*(e*x+d)^4/e+b^3/c*(1/4*c*e^3*arctan(c*x)^3*x^4+c*e^2*arctan(c*x)^3*
x^3*d+3/2*c*e*arctan(c*x)^3*x^2*d^2+arctan(c*x)^3*c*x*d^3+1/4*c/e*arctan(c*
x)^3*d^4-3/4/c^3/e*(2*I*e^3*c*d*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2-I*e*c^3*d^3*Pi*csgn(I*(1+(1+I*c
*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-2
*I*e*c^3*d^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1))^2*arctan(c*x)^2+I*e*c^3*d^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2-I*e*c^3*d^3*Pi*csgn(I/(1+(
1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c
^2*x^2+1))^2)^2*arctan(c*x)^2+2*I*e*c^3*d^3*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x
^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-I*e*c^3*d^3*P
i*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*
x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+I*e^3*c*d*Pi*csgn(I*(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-2*I*e^3*c*
d*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))
^2)^2*arctan(c*x)^2+I*e^3*c*d*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+I
```

```
*e^3*c*d*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/
(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-I*e^3*c*d*Pi*csgn(I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+1/3*e^
4*arctan(c*x)*(c*x-I)^2+arctan(c*x)^3*c^4*d^4+4/3*I*e^4*arctan(c*x)^2-8/3*e
^4*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-8/3*e^4*arctan(c*x)*ln(1
-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+8/3*I*e^4*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(
1/2))+8/3*I*e^4*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^3*e^4-1/
9*(6*c^4*d^4-36*c^2*d^2*e^2+6*e^4)*arctan(c*x)^3-I*e^3*c*d*Pi*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-I*e^3*c*d*
Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2+I*e^3*c*d*Pi*csgn(I*(1+(
1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2+I*e*c^3*d^3*Pi*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1))^3*arctan(c*x)^2+I*e*c^3*d^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-I*e*c^3*d^3*Pi*csgn(I*(1+(
1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2+2*arctan(c*x)^2*ln(c^2*x^2+1)*c^
3*d^3*e-2*arctan(c*x)^2*ln(c^2*x^2+1)*c*d*e^3+12*e^2*d^2*c^2*arctan(c*x)*ln
(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+12*e^2*d^2*c^2*arctan(c*x)*ln(1-I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))-4*e*ln(2)*c^3*d^3*arctan(c*x)^2+4*e^3*ln(2)*c*d*arcta
n(c*x)^2+1/3*arctan(c*x)^2*e^4*c^3*x^3-arctan(c*x)^2*e^4*c*x-6*arctan(c*x)^
3*c^2*d^2*e^2-I*e^3*c*d*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^
2+1))^2)*arctan(c*x)^2+I*e*c^3*d^3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)
*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x
)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-4*e^3*c*d*arctan(c*x)*(c*x-I)-4*e*d^3*c^3
*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)^2+4*e^3*d*c*ln((1+I*c*x)/(c^2*
x^2+1)^(1/2))*arctan(c*x)^2-12*I*e^2*d^2*c^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))-12*I*e^2*d^2*c^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*e^2*d^
2*c^2*arctan(c*x)^2+4/3*I*e*d^3*c^3*arctan(c*x)^3-4/3*I*e^3*d*c*arctan(c*x)
^3+1/3*e^4*(c*x+I)-2*e*c^3*d^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*e^3*c*
d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-4*e^3*c*d*ln(1+(1+I*c*x)^2/(c^2*x^2+1
))+2*e^3*c*d*arctan(c*x)^2+2/3*I*e^4*arctan(c*x)*(c*x-I)-2/3*e^4*arctan(c*x
)*(c*x-I)*(c*x+I)+6*arctan(c*x)^2*c^3*d^2*e^2*x+2*arctan(c*x)^2*e^3*c^3*d*x
^2+4*I*e*c^3*d^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-4*I*e^3*c*
d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))))+3*a*b^2/c*(1/4*c*e^3*ar
ctan(c*x)^2*x^4+c*e^2*arctan(c*x)^2*x^3*d+3/2*c*e*arctan(c*x)^2*x^2*d^2+arc
tan(c*x)^2*c*x*d^3+1/4*c/e*arctan(c*x)^2*d^4-1/2/c^3/e*(6*arctan(c*x)*c^3*d
^2*e^2*x+2*arctan(c*x)*e^3*c^3*d*x^2+1/3*arctan(c*x)*e^4*c^3*x^3-arctan(c*x
)*e^4*c*x+2*arctan(c*x)*ln(c^2*x^2+1)*c^3*d^3*e-2*arctan(c*x)*ln(c^2*x^2+1)
*c*d*e^3+arctan(c*x)^2*c^4*d^4-6*arctan(c*x)^2*c^2*d^2*e^2+arctan(c*x)^2*e^
4-1/12*(6*c^4*d^4-36*c^2*d^2*e^2+6*e^4)*arctan(c*x)^2-1/3*e^2*(6*c^2*d*e*x+
1/2*c^2*e^2*x^2+1/2*(18*c^2*d^2-4*e^2)*ln(c^2*x^2+1)-6*e*arctan(c*x)*c*d)-2
*c*d*e*(c^2*d^2-e^2)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))
-ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)+1/2*I*(ln(c*x+I)*ln(c^2*x^2+
1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2))))+3*a
^2*b/c*(1/4*c*e^3*arctan(c*x)*x^4+c*e^2*arctan(c*x)*x^3*d+3/2*c*e*arctan(c*
x)*x^2*d^2+arctan(c*x)*c*x*d^3+1/4*c/e*arctan(c*x)*d^4-1/4/c^3/e*(6*c^3*d^2
```

```
*e^2*x+2*e^3*c^3*d*x^2+1/3*e^4*c^3*x^3-c*e^4*x+1/2*(4*c^3*d^3*e-4*c*d*e^3)*
ln(c^2*x^2+1)+(c^4*d^4-6*c^2*d^2*e^2+e^4)*arctan(c*x)))
```

Fricas [F]

$$\int (d+ex)^3(a+b\arctan(cx))^3\,dx=\int (ex+d)^3(b\arctan{(cx)}+a)^3\,dx$$

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*e^3*x^3 + 3*a^3*d*e^2*x^2 + 3*a^3*d^2*e*x + a^3*d^3 + (b^3*e^3
*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*arctan(c*x)^3 + 3*(a*b^2*
e^3*x^3 + 3*a*b^2*d*e^2*x^2 + 3*a*b^2*d^2*e*x + a*b^2*d^3)*arctan(c*x)^2 +
3*(a^2*b*e^3*x^3 + 3*a^2*b*d*e^2*x^2 + 3*a^2*b*d^2*e*x + a^2*b*d^3)*arctan(
c*x), x)
```

Sympy [F]

$$\int (d+ex)^3(a+b\arctan(cx))^3\,dx=\int \left(a+b\operatorname{atan}{(cx)}\right)^3\left(d+ex\right)^3\,dx$$

```
[In] integrate((e*x+d)**3*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3*(d + e*x)**3, x)
```

Maxima [F]

$$\int (d+ex)^3(a+b\arctan(cx))^3\,dx=\int (ex+d)^3(b\arctan{(cx)}+a)^3\,dx$$

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*e^3*x^4 + a^3*d*e^2*x^3 + 7/32*b^3*d^3*arctan(c*x)^4/c + 112*b^3*c^
2*e^3*integrate(1/128*x^5*arctan(c*x)^3/(c^2*x^2 + 1), x) + 12*b^3*c^2*e^3*
integrate(1/128*x^5*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 384*
a*b^2*c^2*e^3*integrate(1/128*x^5*arctan(c*x)^2/(c^2*x^2 + 1), x) + 336*b^3
*c^2*d*e^2*integrate(1/128*x^4*arctan(c*x)^3/(c^2*x^2 + 1), x) + 12*b^3*c^2
*e^3*integrate(1/128*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3
6*b^3*c^2*d*e^2*integrate(1/128*x^4*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2
 + 1), x) + 1152*a*b^2*c^2*d*e^2*integrate(1/128*x^4*arctan(c*x)^2/(c^2*x^2
 + 1), x) + 336*b^3*c^2*d^2*e*integrate(1/128*x^3*arctan(c*x)^3/(c^2*x^2 +
1), x) + 48*b^3*c^2*d*e^2*integrate(1/128*x^4*arctan(c*x)*log(c^2*x^2 + 1)/
```

```
(c^2*x^2 + 1), x) + 36*b^3*c^2*d^2*e*integrate(1/128*x^3*arctan(c*x)*log(c^
2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1152*a*b^2*c^2*d^2*e*integrate(1/128*x^3*a
rctan(c*x)^2/(c^2*x^2 + 1), x) + 112*b^3*c^2*d^3*integrate(1/128*x^2*arctan
(c*x)^3/(c^2*x^2 + 1), x) + 72*b^3*c^2*d^2*e*integrate(1/128*x^3*arctan(c*x
)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 12*b^3*c^2*d^3*integrate(1/128*x^2*a
rctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 384*a*b^2*c^2*d^3*integra
te(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 48*b^3*c^2*d^3*integrate(1/1
28*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3/2*a^3*d^2*e*x^2 +
 a*b^2*d^3*arctan(c*x)^3/c - 12*b^3*c*e^3*integrate(1/128*x^4*arctan(c*x)^2
/(c^2*x^2 + 1), x) + 3*b^3*c*e^3*integrate(1/128*x^4*log(c^2*x^2 + 1)^2/(c^
2*x^2 + 1), x) - 48*b^3*c*d*e^2*integrate(1/128*x^3*arctan(c*x)^2/(c^2*x^2
+ 1), x) + 12*b^3*c*d*e^2*integrate(1/128*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 +
 1), x) - 72*b^3*c*d^2*e*integrate(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x
) + 18*b^3*c*d^2*e*integrate(1/128*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
 - 48*b^3*c*d^3*integrate(1/128*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^3*
c*d^3*integrate(1/128*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 9/2*(x^2*arc
tan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b*d^2*e + 3/2*(2*x^3*arctan(c*x
) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a^2*b*d*e^2 + 1/4*(3*x^4*arctan(c*x
) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b*e^3 + a^3*d^3*x + 11
2*b^3*e^3*integrate(1/128*x^3*arctan(c*x)^3/(c^2*x^2 + 1), x) + 12*b^3*e^3*
integrate(1/128*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 384*
a*b^2*e^3*integrate(1/128*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + 336*b^3*d*e
^2*integrate(1/128*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) + 36*b^3*d*e^2*integ
rate(1/128*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1152*a*b^
2*d*e^2*integrate(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 336*b^3*d^2*e
*integrate(1/128*x*arctan(c*x)^3/(c^2*x^2 + 1), x) + 36*b^3*d^2*e*integrate
(1/128*x*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1152*a*b^2*d^2*
e*integrate(1/128*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^3*d^3*integrate(
1/128*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(
c*x) - log(c^2*x^2 + 1))*a^2*b*d^3/c + 1/32*(b^3*e^3*x^4 + 4*b^3*d*e^2*x^3
+ 6*b^3*d^2*e*x^2 + 4*b^3*d^3*x)*arctan(c*x)^3 - 3/128*(b^3*e^3*x^4 + 4*b^3
*d*e^2*x^3 + 6*b^3*d^2*e*x^2 + 4*b^3*d^3*x)*arctan(c*x)*log(c^2*x^2 + 1)^2
```

Giac [F]

$$\int (d+ex)^3(a+b\arctan(cx))^3\,dx = \int (ex+d)^3(b\arctan(cx)+a)^3\,dx$$

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(a+b\arctan(cx))^3\,dx=\int \left(a+b\,\mathrm{atan}(c\,x)\right)^3\,(d+e\,x)^3\,dx$$

```
[In] int((a + b*atan(c*x))^3*(d + e*x)^3,x)
[Out] int((a + b*atan(c*x))^3*(d + e*x)^3, x)
```

3.16 $\int (d+ex)^2(a+b\arctan(cx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 411

$$\begin{aligned}
&\int (d+ex)^2(a+b\arctan(cx))^3\,dx \\
&= \frac{ab^2e^2x}{c^2}+\frac{b^3e^2x\arctan(cx)}{c^2}-\frac{3ibde(a+b\arctan(cx))^2}{c^2} \\
&\quad -\frac{be^2(a+b\arctan(cx))^2}{2c^3}-\frac{3bdex(a+b\arctan(cx))^2}{c} \\
&\quad -\frac{be^2x^2(a+b\arctan(cx))^2}{2c}+\frac{i(3c^2d^2-e^2)\,(a+b\arctan(cx))^3}{3c^3} \\
&\quad -\frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^3}{3e}+\frac{(d+ex)^3(a+b\arctan(cx))^3}{3e} \\
&\quad -\frac{6b^2de(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2}+\frac{b(3c^2d^2-e^2)\,(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} \\
&\quad -\frac{b^3e^2\log\left(1+c^2x^2\right)}{2c^3}-\frac{3ib^3de\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2} \\
&\quad +\frac{ib^2(3c^2d^2-e^2)\,(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} \\
&\quad +\frac{b^3(3c^2d^2-e^2)\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}
\end{aligned}$$

```
[Out] a*b^2*e^2*x/c^2+b^3*e^2*x*arctan(c*x)/c^2-3*I*b*d*e*(a+b*arctan(c*x))^2/c^2
-1/2*b*e^2*(a+b*arctan(c*x))^2/c^3-3*b*d*e*x*(a+b*arctan(c*x))^2/c-1/2*b*e^
2*x^2*(a+b*arctan(c*x))^2/c+1/3*I*(3*c^2*d^2-e^2)*(a+b*arctan(c*x))^3/c^3-1
/3*d*(d^2-3*e^2/c^2)*(a+b*arctan(c*x))^3/e+1/3*(e*x+d)^3*(a+b*arctan(c*x))^
3/e-6*b^2*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2+b*(3*c^2*d^2-e^2)*(a+b*
arctan(c*x))^2*ln(2/(1+I*c*x))/c^3-1/2*b^3*e^2*ln(c^2*x^2+1)/c^3-3*I*b^3*d*
e*polylog(2,1-2/(1+I*c*x))/c^2+I*b^2*(3*c^2*d^2-e^2)*(a+b*arctan(c*x))*poly
```

```
log(2,1-2/(1+I*c*x))/c^3+1/2*b^3*(3*c^2*d^2-e^2)*polylog(3,1-2/(1+I*c*x))/c
^3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {4974, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 5104, 5114, 6745}

$$\int (d+ex)^2(a+b\arctan(cx))^3\,dx$$
$$= -\frac{6b^2de\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^2} + \frac{ib^2(3c^2d^2-e^2)\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^3} - \frac{be^2(a+b\arctan(cx))^2}{2c^3} - \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^3}{3e} - \frac{3ibde(a+b\arctan(cx))^2}{c^2} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^3}{3c^3} + \frac{b(3c^2d^2-e^2)\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^3} - \frac{3bdex(a+b\arctan(cx))^2}{c} + \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e} - \frac{be^2x^2(a+b\arctan(cx))^2}{2c} + \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x\arctan(cx)}{c^2} - \frac{3ib^3de\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2} + \frac{b^3(3c^2d^2-e^2)\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^3} - \frac{b^3e^2\log(c^2x^2+1)}{2c^3}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2*e^2*x)/c^2 + (b^3*e^2*x*ArcTan[c*x])/c^2 - ((3*I)*b*d*e*(a + b*ArcTa
n[c*x])^2)/c^2 - (b*e^2*(a + b*ArcTan[c*x])^2)/(2*c^3) - (3*b*d*e*x*(a + b*
ArcTan[c*x])^2)/c - (b*e^2*x^2*(a + b*ArcTan[c*x])^2)/(2*c) + ((I/3)*(3*c^2
*d^2 - e^2)*(a + b*ArcTan[c*x])^3)/c^3 - (d*(d^2 - (3*e^2)/c^2)*(a + b*ArcT
an[c*x])^3)/(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x])^3)/(3*e) - (6*b^2*d*e*
(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (b*(3*c^2*d^2 - e^2)*(a + b*A
rcTan[c*x])^2*Log[2/(1 + I*c*x)])/c^3 - (b^3*e^2*Log[1 + c^2*x^2])/(2*c^3)
- ((3*I)*b^3*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^2 + (I*b^2*(3*c^2*d^2 - e
^2)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (b^3*(3*c^2*d^
2 - e^2)*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

integral

$$= \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e} - \frac{(bc)\int\left(\frac{3de^2(a+b\arctan(cx))^2}{c^2}+\frac{e^3x(a+b\arctan(cx))^2}{c^2}+\frac{(c^2d^3-3de^2+e(3c^2d^2-e^2)x)(a+b\arctan(cx))^2}{c^2(1+c^2x^2)}\right)dx}{e}$$

$$= \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e} - \frac{b\int\frac{(c^2d^3-3de^2+e(3c^2d^2-e^2)x)(a+b\arctan(cx))^2}{1+c^2x^2}dx}{ce} - \frac{(3bde)\int(a+b\arctan(cx))^2\,dx}{c} - \frac{(be^2)\int x(a+b\arctan(cx))^2\,dx}{c}$$

$$= -\frac{3bdex(a+b\arctan(cx))^2}{c} - \frac{be^2x^2(a+b\arctan(cx))^2}{2c} + \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e}$$
$$- \frac{b\int\left(\frac{c^2d^3\left(1-\frac{3e^2}{c^2d^2}\right)(a+b\arctan(cx))^2}{1+c^2x^2} - \frac{e(-3c^2d^2+e^2)x(a+b\arctan(cx))^2}{1+c^2x^2}\right)dx}{ce}$$
$$+ (6b^2de)\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}dx + (b^2e^2)\int\frac{x^2(a+b\arctan(cx))}{1+c^2x^2}dx$$
$$= -\frac{3ibde(a+b\arctan(cx))^2}{c^2} - \frac{3bdex(a+b\arctan(cx))^2}{c} - \frac{be^2x^2(a+b\arctan(cx))^2}{2c}$$
$$+ \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e} - \frac{(6b^2de)\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}$$
$$+ \frac{(b^2e^2)\int(a+b\arctan(cx))\,dx}{c^2} - \frac{(b^2e^2)\int\frac{a+b\arctan(cx)}{1+c^2x^2}dx}{c^2}$$
$$- \left(bd\left(\frac{cd^2}{e} - \frac{3e}{c}\right)\right)\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}dx - \frac{(b(3c^2d^2-e^2))\int\frac{x(a+b\arctan(cx))^2}{1+c^2x^2}dx}{c}$$
$$= \frac{ab^2e^2x}{c^2} - \frac{3ibde(a+b\arctan(cx))^2}{c^2} - \frac{be^2(a+b\arctan(cx))^2}{2c^3} - \frac{3bdex(a+b\arctan(cx))^2}{c}$$
$$- \frac{be^2x^2(a+b\arctan(cx))^2}{2c} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^3}{3c^3}$$
$$- \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^3}{3e} + \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e}$$
$$- \frac{6b^2de(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} + \frac{(6b^3de)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}dx}{c}$$
$$+ \frac{(b^3e^2)\int\arctan(cx)\,dx}{c^2} + \frac{(b(3c^2d^2-e^2))\int\frac{(a+b\arctan(cx))^2}{i-cx}dx}{c^2}$$
$$= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x\arctan(cx)}{c^2} - \frac{3ibde(a+b\arctan(cx))^2}{c^2} - \frac{be^2(a+b\arctan(cx))^2}{2c^3}$$
$$- \frac{3bdex(a+b\arctan(cx))^2}{c} - \frac{be^2x^2(a+b\arctan(cx))^2}{2c}$$
$$+ \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^3}{3c^3} - \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^3}{3e}$$
$$+ \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e} - \frac{6b^2de(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2}$$
$$+ \frac{b(3c^2d^2-e^2)(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{(6ib^3de)\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1+icx}\right)}{c^2}$$
$$- \frac{(b^3e^2)\int\frac{x}{1+c^2x^2}dx}{c} - \frac{(2b^2(3c^2d^2-e^2))\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}dx}{c^2}$$

$$= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x\arctan(cx)}{c^2} - \frac{3ibde(a+b\arctan(cx))^2}{c^2} - \frac{be^2(a+b\arctan(cx))^2}{2c^3} - \frac{3bdex(a+b\arctan(cx))^2}{c} - \frac{be^2x^2(a+b\arctan(cx))^2}{2c} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^3}{3c^3} - \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^3}{3e} + \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e} - \frac{6b^2de(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} + \frac{b(3c^2d^2-e^2)(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{b^3e^2\log(1+c^2x^2)}{2c^3} - \frac{3ib^3de\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2} + \frac{ib^2(3c^2d^2-e^2)(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} - \frac{(ib^3(3c^2d^2-e^2))\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}dx}{c^2}$$

$$= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x\arctan(cx)}{c^2} - \frac{3ibde(a+b\arctan(cx))^2}{c^2} - \frac{be^2(a+b\arctan(cx))^2}{2c^3} - \frac{3bdex(a+b\arctan(cx))^2}{c} - \frac{be^2x^2(a+b\arctan(cx))^2}{2c} + \frac{i(3c^2d^2-e^2)(a+b\arctan(cx))^3}{3c^3} - \frac{d\left(d^2-\frac{3e^2}{c^2}\right)(a+b\arctan(cx))^3}{3e} + \frac{(d+ex)^3(a+b\arctan(cx))^3}{3e} - \frac{6b^2de(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} + \frac{b(3c^2d^2-e^2)(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{b^3e^2\log(1+c^2x^2)}{2c^3} - \frac{3ib^3de\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2} + \frac{ib^2(3c^2d^2-e^2)(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} + \frac{b^3(3c^2d^2-e^2)\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.51

$$\int (d+ex)^2(a+b\arctan(cx))^3\,dx = \frac{6a^2c^2d(acd-3be)x + 3a^2c^2e(2acd-be)x^2 + 2a^3c^3e^2x^3 + 18a^2bcde\arctan(cx) + 6a^2bc^3x(3d^2+3dex+e^2x}{}$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (6*a^2*c^2*d*(a*c*d - 3*b*e)*x + 3*a^2*c^2*e*(2*a*c*d - b*e)*x^2 + 2*a^3*c^
3*e^2*x^3 + 18*a^2*b*c*d*e*ArcTan[c*x] + 6*a^2*b*c^3*x*(3*d^2 + 3*d*e*x + e
```

```
^2*x^2)*ArcTan[c*x] - 3*a^2*b*(3*c^2*d^2 - e^2)*Log[1 + c^2*x^2] + 18*a*b^2
*c*d*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]
) + 18*a*b^2*c^2*d^2*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2
*I)*ArcTan[c*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + 6*a*b^2*e^2*(c
*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*(1 + c^2*x^2 + 2*Log[1 + E^(
(2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + 6*b^3*c*d*e*
(ArcTan[c*x]*((3*I - 3*c*x)*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 6*L
og[1 + E^((2*I)*ArcTan[c*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])
+ b^3*e^2*(6*c*x*ArcTan[c*x] - 3*ArcTan[c*x]^2 - 3*c^2*x^2*ArcTan[c*x]^2 +
(2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[c*x]^3 - 6*ArcTan[c*x]^2*Log[1 + E^(
(2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2] + (6*I)*ArcTan[c*x]*PolyLog[2, -E^
((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]) + 3*b^3*c^2*d^
2*(2*ArcTan[c*x]^2*((-I + c*x)*ArcTan[c*x] + 3*Log[1 + E^((2*I)*ArcTan[c*x]
)]) - (6*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 3*PolyLog[3, -
E^((2*I)*ArcTan[c*x])]))/(6*c^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.97 (sec) , antiderivative size = 2633, normalized size of antiderivative = 6.41

method	result	size
parts	Expression too large to display	2633
derivativedivides	Expression too large to display	2647
default	Expression too large to display	2647

```
[In] int((e*x+d)^2*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^3*(e*x+d)^3/e+b^3/c*(1/3*c*e^2*arctan(c*x)^3*x^3+c*e*arctan(c*x)^3*x^
2*d+arctan(c*x)^3*c*x*d^2+1/3*c/e*arctan(c*x)^3*d^3-1/c^2/e*(d^3*c^3*arctan
(c*x)^3+1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))*e^3-e^3*ln(1+(1+I*c*x)^2/(c
^2*x^2+1))+1/2*e^3*arctan(c*x)^2+6*e^2*d*c*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))+6*e^2*d*c*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3
*e*d^2*c^2*ln(2)*arctan(c*x)^2-3*e*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))*c^2*d^2*
arctan(c*x)^2+3/2*arctan(c*x)^2*ln(c^2*x^2+1)*e*c^2*d^2-6*I*e^2*d*c*dilog(1
-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/4*I*e^3*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2
+1))^2)^3*arctan(c*x)^2-1/4*I*e^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arct
an(c*x)^2-1/4*I*e^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x
^2+1))^2)^3*arctan(c*x)^2-3*I*c*d*e^2*arctan(c*x)^2-6*I*e^2*d*c*dilog(1+I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))+I*e*c^2*d^2*arctan(c*x)^3+3/2*I*e*d^2*c^2*Pi*cs
gn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*a
rctan(c*x)^2+3/4*I*e*d^2*c^2*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(
I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2-3/2*I*e*d^2*c^2*Pi*csgn(I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2-3/4*I*
```

e*d^2*c^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-3/4*I*e*d^2*c^2*Pi*csgn(I/(
1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2
/(c^2*x^2+1))^2)^2*arctan(c*x)^2-3/4*I*e*d^2*c^2*Pi*csgn(I*(1+(1+I*c*x)^2/(
c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-1/2*arct
an(c*x)^2*ln(c^2*x^2+1)*e^3-1/3*I*e^3*arctan(c*x)^3-e^3*arctan(c*x)*(c*x-I)
+ln((1+I*c*x)/(c^2*x^2+1)^(1/2))*e^3*arctan(c*x)^2+e^3*ln(2)*arctan(c*x)^2-
3*e^2*d*c*arctan(c*x)^3+1/2*arctan(c*x)^2*e^3*c^2*x^2-2/3*d*c*(c^2*d^2-3*e^
2)*arctan(c*x)^3-I*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*e^3*arctan(c*x)-3/2*
e*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))*c^2*d^2+3*arctan(c*x)^2*c^2*d*e^2*x-1
/2*I*e^3*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*
x^2+1))^2)^2*arctan(c*x)^2+1/4*I*e^3*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))
^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2+1/4*I*e^3*Pi*csgn(I/
(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^
2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+1/2*I*e^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2+1/4*I*e^3*Pi*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^
2*x^2+1))^2)^2*arctan(c*x)^2-1/4*I*e^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+3*I*e*polylog(2,-(1+I*c*
x)^2/(c^2*x^2+1))*c^2*d^2*arctan(c*x)-1/4*I*e^3*Pi*csgn(I/(1+(1+I*c*x)^2/(c
^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2+3/4*I*e*d^2*c^2*Pi*csgn(I*(1
+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2+3/4*I*e*d^2*c^2*Pi*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-3/4*I*e*d^2*c^
2*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2+3/4*I*e*d^2*c^2*
Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2))
+3*a*b^2/c*(1/3*c*e^2*arctan(c*x)^2*x^3+c*e*arctan(c*x)^2*x^2*d+arctan(c*x)
^2*c*x*d^2+1/3*c/e*arctan(c*x)^2*d^3-2/3/c^2/e*(3*arctan(c*x)*c^2*d*e^2*x+1
/2*arctan(c*x)*e^3*c^2*x^2+3/2*arctan(c*x)*ln(c^2*x^2+1)*e*c^2*d^2-1/2*arct
an(c*x)*ln(c^2*x^2+1)*e^3+arctan(c*x)^2*c^3*d^3-3*arctan(c*x)^2*c*d*e^2-1/2
e(3*c^2*d^2-e^2)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-l
n(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)
-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2))-3/2*e^2
*ln(c^2*x^2+1)*c*d+1/2*e^3*arctan(c*x)-1/2*c*x*e^3-1/2*d*c*(c^2*d^2-3*e^2)*
arctan(c*x)^2))+a^2*b*e^2*arctan(c*x)*x^3+3*a^2*b*e*arctan(c*x)*x^2*d+3*a^2
*b*arctan(c*x)*x*d^2-1/2/c*e^2*a^2*b*x^2-3/c*d*e*x*a^2*b-3/2*a^2*b/c*ln(c^2
*x^2+1)*d^2+1/2*a^2*b/c^3*e^2*ln(c^2*x^2+1)+3*a^2*b/c^2*e*arctan(c*x)*d

Fricas [F]

$$\int (d+ex)^2(a+b\arctan(cx))^3\,dx = \int (ex+d)^2(b\arctan(cx)+a)^3\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*e^2*x^2 + 2*a^3*d*e*x + a^3*d^2 + (b^3*e^2*x^2 + 2*b^3*d*e*x +
 b^3*d^2)*arctan(c*x)^3 + 3*(a*b^2*e^2*x^2 + 2*a*b^2*d*e*x + a*b^2*d^2)*arc
tan(c*x)^2 + 3*(a^2*b*e^2*x^2 + 2*a^2*b*d*e*x + a^2*b*d^2)*arctan(c*x), x)
```

Sympy [F]

$$\int (d+ex)^2(a+b\arctan(cx))^3\,dx = \int (a+b\,\mathrm{atan}(cx))^3\,(d+ex)^2\,dx$$

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3*(d + e*x)**2, x)
```

Maxima [F]

$$\int (d+ex)^2(a+b\arctan(cx))^3\,dx = \int (ex+d)^2(b\arctan(cx)+a)^3\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*e^2*x^3 + 7/32*b^3*d^2*arctan(c*x)^4/c + 28*b^3*c^2*e^2*integrate(1
/32*x^4*arctan(c*x)^3/(c^2*x^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*x^4*
arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 96*a*b^2*c^2*e^2*integra
te(1/32*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) + 56*b^3*c^2*d*e*integrate(1/32
*x^3*arctan(c*x)^3/(c^2*x^2 + 1), x) + 4*b^3*c^2*e^2*integrate(1/32*x^4*arc
tan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 6*b^3*c^2*d*e*integrate(1/32*
x^3*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*d*e*in
tegrate(1/32*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + 28*b^3*c^2*d^2*integrate
(1/32*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) + 12*b^3*c^2*d*e*integrate(1/32*x
^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*b^3*c^2*d^2*integrate
(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 96*a*b^2*c^2*d
^2*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^3*c^2*d^2*inte
grate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + a^3*d*e*x^2
 + a*b^2*d^2*arctan(c*x)^3/c - 4*b^3*c*e^2*integrate(1/32*x^3*arctan(c*x)^2
/(c^2*x^2 + 1), x) + b^3*c*e^2*integrate(1/32*x^3*log(c^2*x^2 + 1)^2/(c^2*x
```

```
^2 + 1), x) - 12*b^3*c*d*e*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2 + 1),
x) + 3*b^3*c*d*e*integrate(1/32*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) -
12*b^3*c*d^2*integrate(1/32*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*d^2
*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3*(x^2*arctan(c*x)
 - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b*d*e + 1/2*(2*x^3*arctan(c*x) - c*(x^2
/c^2 - log(c^2*x^2 + 1)/c^4))*a^2*b*e^2 + a^3*d^2*x + 28*b^3*e^2*integrate(
1/32*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) + 3*b^3*e^2*integrate(1/32*x^2*arc
tan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 96*a*b^2*e^2*integrate(1/32
*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 56*b^3*d*e*integrate(1/32*x*arctan(c
*x)^3/(c^2*x^2 + 1), x) + 6*b^3*d*e*integrate(1/32*x*arctan(c*x)*log(c^2*x^
2 + 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*d*e*integrate(1/32*x*arctan(c*x)^2/(
c^2*x^2 + 1), x) + 3*b^3*d^2*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/
(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b*d^2/c
+ 1/24*(b^3*e^2*x^3 + 3*b^3*d*e*x^2 + 3*b^3*d^2*x)*arctan(c*x)^3 - 1/32*(b^
3*e^2*x^3 + 3*b^3*d*e*x^2 + 3*b^3*d^2*x)*arctan(c*x)*log(c^2*x^2 + 1)^2
```

Giac [F]

$$\int (d+ex)^2(a+b\arctan(cx))^3\,dx = \int (ex+d)^2(b\arctan(cx)+a)^3\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(a+b\arctan(cx))^3\,dx = \int (a+b\,\mathrm{atan}(c\,x))^3\,(d+e\,x)^2\,dx$$

```
[In] int((a + b*atan(c*x))^3*(d + e*x)^2,x)
[Out] int((a + b*atan(c*x))^3*(d + e*x)^2, x)
```

3.17 $\int (d+ex)(a+b\arctan(cx))^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 264

$$\begin{aligned}\int (d+ex)(a+b\arctan(cx))^3\,dx &= -\frac{3ibe(a+b\arctan(cx))^2}{2c^2}-\frac{3bex(a+b\arctan(cx))^2}{2c}\\
&+\frac{id(a+b\arctan(cx))^3}{c}-\frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^3}{2e}\\
&+\frac{(d+ex)^2(a+b\arctan(cx))^3}{2e}\\
&-\frac{3b^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2}\\
&+\frac{3bd(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c}\\
&-\frac{3ib^3e\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}\\
&+\frac{3ib^2d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}\\
&+\frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}\end{aligned}$$

```
[Out] -3/2*I*b*e*(a+b*arctan(c*x))^2/c^2-3/2*b*e*x*(a+b*arctan(c*x))^2/c+I*d*(a+b
*arctan(c*x))^3/c-1/2*(d^2-e^2/c^2)*(a+b*arctan(c*x))^3/e+1/2*(e*x+d)^2*(a+
b*arctan(c*x))^3/e-3*b^2*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2+3*b*d*(a+b
*arctan(c*x))^2*ln(2/(1+I*c*x))/c-3/2*I*b^3*e*polylog(2,1-2/(1+I*c*x))/c^2+
3*I*b^2*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3*d*polylog(3,
1-2/(1+I*c*x))/c
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4974, 4930, 5040, 4964, 2449, 2352, 5104, 5004, 5114, 6745}

$$\int (d+ex)(a+b\arctan(cx))^3\,dx = -\frac{3b^2e\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^2} + \frac{3ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^3}{2e} - \frac{3ibe(a+b\arctan(cx))^2}{2c^2} + \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} + \frac{id(a+b\arctan(cx))^3}{c} + \frac{3bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c} - \frac{3bex(a+b\arctan(cx))^2}{2c} - \frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^2} + \frac{3b^3d\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c}$$

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c*x])^2)/c^2 - (3*b*e*x*(a + b*ArcTan[c*x])^2
)/(2*c) + (I*d*(a + b*ArcTan[c*x])^3)/c - ((d^2 - e^2/c^2)*(a + b*ArcTan[c*
x])^3)/(2*e) + ((d + e*x)^2*(a + b*ArcTan[c*x])^3)/(2*e) - (3*b^2*e*(a + b*
ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (3*b*d*(a + b*ArcTan[c*x])^2*Log[2/(
1 + I*c*x)])/c - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^2 + ((3*
I)*b^2*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + (3*b^3*d*Po
lyLog[3, 1 - 2/(1 + I*c*x)])/(2*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} \\
&\quad - \frac{(3bc)\int\left(\frac{e^2(a+b\arctan(cx))^2}{c^2}+\frac{(c^2d^2-e^2+2c^2dex)(a+b\arctan(cx))^2}{c^2(1+c^2x^2)}\right)dx}{2e} \\
&= \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} - \frac{(3b)\int\frac{(c^2d^2-e^2+2c^2dex)(a+b\arctan(cx))^2}{1+c^2x^2}dx}{2ce} \\
&\quad - \frac{(3be)\int(a+b\arctan(cx))^2\,dx}{2c} \\
&= -\frac{3bex(a+b\arctan(cx))^2}{2c} + \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} \\
&\quad - \frac{(3b)\int\left(\frac{c^2d^2\left(1-\frac{e^2}{c^2d^2}\right)(a+b\arctan(cx))^2}{1+c^2x^2}+\frac{2c^2dex(a+b\arctan(cx))^2}{1+c^2x^2}\right)dx}{2ce} \\
&\quad + \left(3b^2e\right)\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx \\
&= -\frac{3ibe(a+b\arctan(cx))^2}{2c^2} - \frac{3bex(a+b\arctan(cx))^2}{2c} \\
&\quad + \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} - (3bcd)\int\frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx \\
&\quad - \frac{\left(3b^2e\right)\int\frac{a+b\arctan(cx)}{i-cx}dx}{c} - \frac{(3b(cd-e)(cd+e))\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}dx}{2ce} \\
&= -\frac{3ibe(a+b\arctan(cx))^2}{2c^2} - \frac{3bex(a+b\arctan(cx))^2}{2c} \\
&\quad + \frac{id(a+b\arctan(cx))^3}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^3}{2e} \\
&\quad + \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} - \frac{3b^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} \\
&\quad + (3bd)\int\frac{(a+b\arctan(cx))^2}{i-cx}\,dx + \frac{\left(3b^3e\right)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}dx}{c}
\end{aligned}$$

$$= -\frac{3ibe(a+b\arctan(cx))^2}{2c^2} - \frac{3bex(a+b\arctan(cx))^2}{2c} + \frac{id(a+b\arctan(cx))^3}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^3}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} - \frac{3b^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} + \frac{3bd(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c} - (6b^2d)\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx - \frac{(3ib^3e)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{c^2}$$

$$= -\frac{3ibe(a+b\arctan(cx))^2}{2c^2} - \frac{3bex(a+b\arctan(cx))^2}{2c} + \frac{id(a+b\arctan(cx))^3}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^3}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} - \frac{3b^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} + \frac{3bd(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c} - \frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2} + \frac{3ib^2d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} - (3ib^3d)\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx$$

$$= -\frac{3ibe(a+b\arctan(cx))^2}{2c^2} - \frac{3bex(a+b\arctan(cx))^2}{2c} + \frac{id(a+b\arctan(cx))^3}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)(a+b\arctan(cx))^3}{2e} + \frac{(d+ex)^2(a+b\arctan(cx))^3}{2e} - \frac{3b^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} + \frac{3bd(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c} - \frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2} + \frac{3ib^2d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} + \frac{3b^3d\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.30

$$\int (d+ex)(a+b\arctan(cx))^3\,dx$$

$$= \frac{a^2c(2acd-3be)x + a^3c^2ex^2 + 3a^2be\arctan(cx) + 3a^2bc^2x(2d+ex)\arctan(cx) - 3a^2bcd\log\left(1+c^2x^2\right) +}{}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a^2*c*(2*a*c*d - 3*b*e)*x + a^3*c^2*e*x^2 + 3*a^2*b*e*ArcTan[c*x] + 3*a^2*
b*c^2*x*(2*d + e*x)*ArcTan[c*x] - 3*a^2*b*c*d*Log[1 + c^2*x^2] + 3*a*b^2*e*
(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]) + 6*a
*b^2*c*d*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c
*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + b^3*e*(ArcTan[c*x]*((3*I -
 3*c*x)*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 6*Log[1 + E^((2*I)*ArcT
an[c*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + b^3*c*d*(2*ArcTan[
c*x]^2*((-I + c*x)*ArcTan[c*x] + 3*Log[1 + E^((2*I)*ArcTan[c*x])]) - (6*I)*
ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 3*PolyLog[3, -E^((2*I)*Arc
Tan[c*x])]))/(2*c^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.93 (sec) , antiderivative size = 3886, normalized size of antiderivative = 14.72

method	result	size
parts	Expression too large to display	3886
derivativedivides	Expression too large to display	3901
default	Expression too large to display	3901

```
[In] int((e*x+d)*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/2*e*x^2+d*x)+b^3/c*(1/2*arctan(c*x)^3*c*x^2*e+arctan(c*x)^3*c*x*d-3/
2/c*(arctan(c*x)^2*c*x*e-c*d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-2*d*c*ln((
1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)^2-2*ln(2)*c*d*arctan(c*x)^2+2*c*d*l
n(2)*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*c*d*ln(2)*dilog(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-c*d*ln(2)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+ln(c^2*x^
2+1)*arctan(c*x)^2*c*d+e*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+e*
arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+e*arctan(c*x)*ln(1+(1+I*c*x
)^2/(c^2*x^2+1))-1/3*arctan(c*x)^3*e-1/4*I*d*c*Pi*csgn(I*(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*(2*I*arctan(c*x)*ln(1+(1
+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))
-1/4*I*d*c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x
^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))-1/4*I*d*c*Pi*cs
gn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*
c*x)^2/(c^2*x^2+1))^2)^2*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*a
rctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))+1/2*I*d*c*Pi*csgn(I*(1+(1
+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*(I*arctan(c
*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(
c^2*x^2+1)^(1/2)))+1/2*I*d*c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*(I*arctan(c*x)*ln(1+I*
```

(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^
(1/2)))+1/4*I*d*c*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)
^2/(c^2*x^2+1))*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x
)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))-1/2*I*d*c*Pi*csgn(I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*(2*I*arctan(c*x)*ln(1+(1+I
*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))-1
/2*I*d*c*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x
^2+1))*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(
1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dil
og(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))+1/2*I*d*c*Pi*csgn(I/(1+(1+I*c*x)^2/(c^
2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^
2*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-
I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))+I*d*c*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^
2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))-I*d*c*Pi*
csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2
*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1
+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I
*(1+I*c*x)/(c^2*x^2+1)^(1/2)))+1/2*I*d*c*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+
1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*(2*I*arctan(c*x)*ln(1+(1+I*c*x
)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))+1/4*I
*d*c*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*(2*I*arcta
n(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1)))-1/2*I*d*c*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1
+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x
^2+1))^2)*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*
ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+
dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))-I*arctan(c*x)^2*e-I*e*dilog(1+I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))-I*e*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*
e*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*d*c*Pi*csgn(I*(1+I*c*x)^2/(c^2*
x^2+1))^3*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*
ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+
dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))+1/4*I*d*c*Pi*csgn(I*(1+I*c*x)^2/(c^
2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2
/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))+1/4*I*d*
c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c
^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))+2*I*ln(2)*c
*d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*ln(2)*c*d*arctan(c*x
)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*ln(2)*c*d*arctan(c*x)*ln(1+(1+I*c
*x)^2/(c^2*x^2+1))+1/2*I*d*c*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*(I*
arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c

```
*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*(1+
I*c*x)/(c^2*x^2+1)^(1/2)))-1/4*I*d*c*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^
2)^3*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog
(2,-(1+I*c*x)^2/(c^2*x^2+1)))-1/2*I*d*c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(
1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))+2*I*d*c*arcta
n(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2/3*I*c*d*arctan(c*x)^3))+3*a*b^
2/c*(1/2*arctan(c*x)^2*c*x^2*e+arctan(c*x)^2*c*x*d-1/c*(ln(c^2*x^2+1)*arcta
n(c*x)*c*d-1/2*arctan(c*x)^2*e+arctan(c*x)*e*c*x-1/2*e*ln(c^2*x^2+1)-d*c*(-
1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c
*x+I))-1/2*ln(c*x-I)^2)+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))
-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2))))+3*a^2*b/c*(1/2*c*arctan(c*
x)*x^2*e+arctan(c*x)*c*x*d-1/2/c*(c*e*x+d*c*ln(c^2*x^2+1)-e*arctan(c*x)))
```

Fricas [F]

$$\int (d+ex)(a+b\arctan(cx))^3\,dx = \int (ex+d)(b\arctan(cx)+a)^3\,dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*e*x + a^3*d + (b^3*e*x + b^3*d)*arctan(c*x)^3 + 3*(a*b^2*e*x +
 a*b^2*d)*arctan(c*x)^2 + 3*(a^2*b*e*x + a^2*b*d)*arctan(c*x), x)
```

Sympy [F]

$$\int (d+ex)(a+b\arctan(cx))^3\,dx = \int (a+b\operatorname{atan}(cx))^3\,(d+ex)\,dx$$

```
[In] integrate((e*x+d)*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex)(a+b\arctan(cx))^3\,dx = \int (ex+d)(b\arctan(cx)+a)^3\,dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 7/32*b^3*d*arctan(c*x)^4/c + 56*b^3*c^2*e*integrate(1/64*x^3*arctan(c*x)^3/
(c^2*x^2 + 1), x) + 6*b^3*c^2*e*integrate(1/64*x^3*arctan(c*x)*log(c^2*x^2
+ 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*e*integrate(1/64*x^3*arctan(c*x)^2
/(c^2*x^2 + 1), x) + 56*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)^3/(c^2*x^2
 + 1), x) + 12*b^3*c^2*e*integrate(1/64*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c
^2*x^2 + 1), x) + 6*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 +
1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*d*integrate(1/64*x^2*arctan(c*x)^2/(
c^2*x^2 + 1), x) + 24*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2
+ 1)/(c^2*x^2 + 1), x) + 1/2*a^3*e*x^2 + a*b^2*d*arctan(c*x)^3/c - 12*b^3*c
*e*integrate(1/64*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*e*integrate
(1/64*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 24*b^3*c*d*integrate(1/64*
x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 6*b^3*c*d*integrate(1/64*x*log(c^2*x^2
+ 1)^2/(c^2*x^2 + 1), x) + 3/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^
3))*a^2*b*e + a^3*d*x + 56*b^3*e*integrate(1/64*x*arctan(c*x)^3/(c^2*x^2 +
1), x) + 6*b^3*e*integrate(1/64*x*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 +
 1), x) + 192*a*b^2*e*integrate(1/64*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 6*
b^3*d*integrate(1/64*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2
*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b*d/c + 1/16*(b^3*e*x^2 + 2*b^3
*d*x)*arctan(c*x)^3 - 3/64*(b^3*e*x^2 + 2*b^3*d*x)*arctan(c*x)*log(c^2*x^2
+ 1)^2
```

Giac [F]

$$\int (d+ex)(a+b\arctan(cx))^3\,dx=\int (ex+d)(b\arctan(cx)+a)^3\,dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)(a+b\arctan(cx))^3\,dx=\int (a+b\,\mathrm{atan}(c\,x))^3\,(d+e\,x)\,dx$$

```
[In] int((a + b*atan(c*x))^3*(d + e*x),x)
```

```
[Out] int((a + b*atan(c*x))^3*(d + e*x), x)
```

3.18 $\int \frac{(a+b\arctan(cx))^3}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 320

$$
\begin{aligned}
\int \frac{(a+b\arctan(cx))^3}{d+ex}\,dx = & -\frac{(a+b\arctan(cx))^3\log\left(\frac{2}{1-icx}\right)}{e} \\
& + \frac{(a+b\arctan(cx))^3\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} \\
& + \frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e} \\
& - \frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e} \\
& - \frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e} \\
& + \frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e} \\
& - \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-icx}\right)}{4e} \\
& + \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{4e}
\end{aligned}
$$

```
[Out] -(a+b*arctan(c*x))^3*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))^3*ln(2*c*(e*x+d)/(
c*d+I*e)/(1-I*c*x))/e+3/2*I*b*(a+b*arctan(c*x))^2*polylog(2,1-2/(1-I*c*x))/
e-3/2*I*b*(a+b*arctan(c*x))^2*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/
e-3/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1-I*c*x))/e+3/2*b^2*(a+b*arctan(
c*x))*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e-3/4*I*b^3*polylog(4,1-
2/(1-I*c*x))/e+3/4*I*b^3*polylog(4,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4970}

$$\int \frac{(a+b\arctan(cx))^3}{d+ex}\,dx = \frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e} - \frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{2e} - \frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e} + \frac{(a+b\arctan(cx))^3\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e} + \frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{2e} - \frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^3}{e} + \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{4e} - \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-icx}\right)}{4e}$$

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])^3*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^3*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (((3*I)/2)*b*(a + b*ArcTa
n[c*x])^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (((3*I)/2)*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (3*b^2*
(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e) + (3*b^2*(a + b*Ar
cTan[c*x])*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e)
 - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*c*x)])/e + (((3*I)/4)*b^3*PolyLog
[4, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^3)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^3*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[3
*I*b*(a + b*ArcTan[c*x])^2*(PolyLog[2, 1 - 2/(1 - I*c*x)]/(2*e)), x] - Simp
[3*I*b*(a + b*ArcTan[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1
 - I*c*x)))]/(2*e)), x] - Simp[3*b^2*(a + b*ArcTan[c*x])*(PolyLog[3, 1 - 2/
(1 - I*c*x)]/(2*e)), x] + Simp[3*b^2*(a + b*ArcTan[c*x])*(PolyLog[3, 1 - 2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x] - Simp[3*I*b^3*(PolyLog
```

```
[4, 1 - 2/(1 - I*c*x)]/(4*e)), x] + Simp[3*I*b^3*(PolyLog[4, 1 - 2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} = &-\frac{(a+b\arctan(cx))^3\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\arctan(cx))^3\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}\\
&+\frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}\\
&-\frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}\\
&-\frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e}\\
&+\frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}\\
&-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-icx}\right)}{4e}+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{4e}
\end{aligned}$$

Mathematica [F(-1)]

Timed out.

$$\int\frac{(a+b\arctan(cx))^3}{d+ex}\,dx=\$\text{Aborted}$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x),x]
[Out] $Aborted
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.20 (sec) , antiderivative size = 2398, normalized size of antiderivative = 7.49

method	result	size
derivativedivides	Expression too large to display	2398
default	Expression too large to display	2398
parts	Expression too large to display	2405

```
[In] int((a+b*arctan(c*x))^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^3*c*ln(c*e*x+c*d)/e+b^3*c*(ln(c*e*x+c*d)/e*arctan(c*x)^3-3/e*(1/3*ar
ctan(c*x)^3*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e
+c*d)-1/6*I*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^
2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*(csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x
^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+
1)))-csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c
*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I
*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I
*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I
*c*x)^2/(c^2*x^2+1)))+csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/
(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2)*arctan(c*x)^3-1/2*I*ar
ctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*arctan(c*x)*polylog(3,-
(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))-1/3*c*d/
(c*d-I*e)*arctan(c*x)^3*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1
/2*c*d/(c*d-I*e)*arctan(c*x)*polylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2
*x^2+1))+1/2*I*c*d/(c*d-I*e)*arctan(c*x)^2*polylog(2,(I*e-c*d)/(c*d+I*e)*(1
+I*c*x)^2/(c^2*x^2+1))-1/4*I*c*d/(c*d-I*e)*polylog(4,(I*e-c*d)/(c*d+I*e)*(1
+I*c*x)^2/(c^2*x^2+1))-1/3*e*arctan(c*x)^3*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*
x)^2/(c^2*x^2+1))/(e+I*d*c)-1/2*e*arctan(c*x)*polylog(3,(I*e-c*d)/(c*d+I*e)
*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+1/2*I*e*arctan(c*x)^2*polylog(2,(I*e-c*
d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)-1/4*I*e*polylog(4,(I*e-c*d)
/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)))+3*a*b^2*c*(ln(c*e*x+c*d)/e*
arctan(c*x)^2-2/e*(1/2*arctan(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1
+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)-1/4*I*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1
)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*(csgn(I
*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I
/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I
*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I/(1+(1+I*c*
x)^2/(c^2*x^2+1)))-csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^
2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^
2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I*(-I*e*(1+I*c*x)^2/(c^
2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^
2)*arctan(c*x)^2-1/2*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*
polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*c*d/(c*d-I*e)*arctan(c*x)*polylog
(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/2*c*d/(c*d-I*e)*arctan(c*
x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/4*c*d/(c*d-I*e)*po
lylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*e*arctan(c*x)*po
lylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)-1/2*e*arctan
(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)-1/4*e*p
olylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)))+3*a^2*b*c
*(ln(c*e*x+c*d)/e*arctan(c*x)-1/2*I*ln(c*e*x+c*d)*(-ln((I*e-c*e*x)/(c*d+I*e
))+ln((I*e+c*e*x)/(I*e-c*d)))/e+1/2*I*(dilog((I*e-c*e*x)/(c*d+I*e))-dilog((
I*e+c*e*x)/(I*e-c*d)))/e))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^3}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d),x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+ex}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^3}{d+ex}\,dx$$

```
[In] integrate((a+b*atan(c*x))**3/(e*x+d),x)
[Out] Integral((a + b*atan(c*x))**3/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^3}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d),x, algorithm="maxima")
[Out] a^3*log(e*x + d)/e + integrate(1/32*(28*b^3*arctan(c*x)^3 + 3*b^3*arctan(c*
x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*arctan(c*x)^2 + 96*a^2*b*arctan(c*x))/(e*x
 + d), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+ex}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{d+e\,x}\,dx$$

```
[In] int((a + b*atan(c*x))^3/(d + e*x),x)
[Out] int((a + b*atan(c*x))^3/(d + e*x), x)
```

3.19 $\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 499

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx &= \frac{ic(a+b\arctan(cx))^3}{c^2d^2+e^2}+\frac{c^2d(a+b\arctan(cx))^3}{e\,(c^2d^2+e^2)}\\
&-\frac{(a+b\arctan(cx))^3}{e(d+ex)}-\frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2}\\
&+\frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2+e^2}\\
&+\frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2}\\
&+\frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2}\\
&+\frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2+e^2}\\
&-\frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2}\\
&-\frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2\,(c^2d^2+e^2)}+\frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2\,(c^2d^2+e^2)}\\
&+\frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\,(c^2d^2+e^2)}\end{aligned}$$

```
[Out] I*c*(a+b*arctan(c*x))^3/(c^2*d^2+e^2)+c^2*d*(a+b*arctan(c*x))^3/e/(c^2*d^2+
e^2)-(a+b*arctan(c*x))^3/e/(e*x+d)-3*b*c*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x)
)/(c^2*d^2+e^2)+3*b*c*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/(c^2*d^2+e^2)+3*b
*c*(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d^2+e^2)+3*
```

```
I*b^2*c*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/(c^2*d^2+e^2)+3*I*b^2*c*
(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/(c^2*d^2+e^2)-3*I*b^2*c*(a+b*arc
tan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d^2+e^2)-3/2*b^
3*c*polylog(3,1-2/(1-I*c*x))/(c^2*d^2+e^2)+3/2*b^3*c*polylog(3,1-2/(1+I*c*x
))/(c^2*d^2+e^2)+3/2*b^3*c*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^
2*d^2+e^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4974, 4968, 5104, 5004, 5040, 4964, 5114, 6745}

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx = \frac{3ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c^2d^2+e^2} + \frac{3ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^2d^2+e^2} - \frac{3ib^2c(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{ic(a+b\arctan(cx))^3}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^3}{e\left(c^2d^2+e^2\right)} - \frac{3bc\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{c^2d^2+e^2} + \frac{3bc\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{c^2d^2+e^2} - \frac{(a+b\arctan(cx))^3}{e(d+ex)} - \frac{3b^3c\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)} + \frac{3b^3c\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2\left(c^2d^2+e^2\right)} + \frac{3b^3c\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)}$$

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + e*x)^2,x]
```

```
[Out] (I*c*(a + b*ArcTan[c*x])^3)/(c^2*d^2 + e^2) + (c^2*d*(a + b*ArcTan[c*x])^3)
/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])^3/(e*(d + e*x)) - (3*b*c*(a + b*
ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (3*b*c*(a + b*ArcTan[c
*x])^2*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2) + (3*b*c*(a + b*ArcTan[c*x])^2*L
og[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) + ((3*I)*b^2
```

```
*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2) + ((3
*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2
) - ((3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d +
 I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) - (3*b^3*c*PolyLog[3, 1 - 2/(1 - I*c*x
)])/(2*(c^2*d^2 + e^2)) + (3*b^3*c*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*(c^2*d
^2 + e^2)) + (3*b^3*c*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*(c^2*d^2 + e^2))
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\arctan(cx))^3}{e(d+ex)} + \frac{(3bc)\int\left(\frac{e^2(a+b\arctan(cx))^2}{(c^2d^2+e^2)(d+ex)} + \frac{c^2(d-ex)(a+b\arctan(cx))^2}{(c^2d^2+e^2)(1+c^2x^2)}\right)dx}{e}$$

$$= -\frac{(a+b\arctan(cx))^3}{e(d+ex)} + \frac{(3bc^3)\int\frac{(d-ex)(a+b\arctan(cx))^2}{1+c^2x^2}dx}{e(c^2d^2+e^2)} + \frac{(3bce)\int\frac{(a+b\arctan(cx))^2}{d+ex}dx}{c^2d^2+e^2}$$

$$= -\frac{(a+b\arctan(cx))^3}{e(d+ex)} - \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} - \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} - \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)} + \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)} + \frac{(3bc^3)\int\left(\frac{d(a+b\arctan(cx))^2}{1+c^2x^2} - \frac{ex(a+b\arctan(cx))^2}{1+c^2x^2}\right)dx}{e(c^2d^2+e^2)}$$

$$= -\frac{(a+b\arctan(cx))^3}{e(d+ex)} - \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} - \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} - \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)} + \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)} - \frac{(3bc^3)\int\frac{x(a+b\arctan(cx))^2}{1+c^2x^2}dx}{c^2d^2+e^2} + \frac{(3bc^3d)\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}dx}{e(c^2d^2+e^2)}$$

$$= \frac{ic(a+b\arctan(cx))^3}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^3}{e(c^2d^2+e^2)} - \frac{(a+b\arctan(cx))^3}{e(d+ex)} - \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} - \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} - \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)} + \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)} + \frac{(3bc^2)\int\frac{(a+b\arctan(cx))^2}{i-cx}dx}{c^2d^2+e^2}$$

$$= \frac{ic(a+b\arctan(cx))^3}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^3}{e\left(c^2d^2+e^2\right)} - \frac{(a+b\arctan(cx))^3}{e(d+ex)} - \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} - \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} - \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)} + \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)} - \frac{\left(6b^2c^2\right)\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2d^2+e^2}$$

$$= \frac{ic(a+b\arctan(cx))^3}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^3}{e\left(c^2d^2+e^2\right)} - \frac{(a+b\arctan(cx))^3}{e(d+ex)} - \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2+e^2} - \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} - \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)} + \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)} - \frac{\left(3ib^3c^2\right)\int \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2d^2+e^2}$$

$$= \frac{ic(a+b\arctan(cx))^3}{c^2d^2+e^2} + \frac{c^2d(a+b\arctan(cx))^3}{e(c^2d^2+e^2)} - \frac{(a+b\arctan(cx))^3}{e(d+ex)} - \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2+e^2} + \frac{3bc(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} + \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2+e^2} - \frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2d^2+e^2} - \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)} + \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2(c^2d^2+e^2)} + \frac{3b^3c\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)}$$

Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx = \int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^2,x]
[Out] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^2, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 22.85 (sec) , antiderivative size = 2398, normalized size of antiderivative = 4.81

method	result	size
derivativedivides	Expression too large to display	2398
default	Expression too large to display	2398
parts	Expression too large to display	2406

```
[In] int((a+b*arctan(c*x))^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
[Out] 1/c*(-a^3*c^2/(c*e*x+c*d)/e+b^3*c^2*(-1/(c*e*x+c*d)/e*arctan(c*x)^3+3/e*(ar
ctan(c*x)^2*e/(c^2*d^2+e^2)*ln(c*e*x+c*d)-1/2*arctan(c*x)^2/(c^2*d^2+e^2)*e
```

```
*ln(c^2*x^2+1)+1/3*arctan(c*x)^3/(c^2*d^2+e^2)*d*c+e/(c^2*d^2+e^2)*arctan(c
*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-e/(c^2*d^2+e^2)*arctan(c*x)^2*ln(-I*e
*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)-I/(c^2*d^2+e^
2)*e*c*d/(c*d-I*e)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c
^2*x^2+1))+1/(c^2*d^2+e^2)*e*c*d/(c*d-I*e)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*
d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2/(c^2*d^2+e^2)*e*c*d/(c*d-I*e)*polylog(3
,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-I*e^2*arctan(c*x)*polylog(2,(
I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(c^2*d^2+e^2)/(e+I*d*c)+e^2*arc
tan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(c^2*d^2+e^2)/
(e+I*d*c)+1/2*e^2*polylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(c
^2*d^2+e^2)/(e+I*d*c)-1/3*I*e/(c^2*d^2+e^2)*arctan(c*x)^3+1/4*e/(c^2*d^2+e^
2)*(I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+
(1+I*c*x)^2/(c^2*x^2+1))^2)^2+2*I*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c
*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+I*Pi*csg
n(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)+I*
Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+
(1+I*c*x)^2/(c^2*x^2+1))^2)^2-I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1))-2*I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*cs
gn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2-I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1
))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+
I*c*x)^2/(c^2*x^2+1))^2)+I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3-2*I*P
i*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+
c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-I*Pi*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3+2*I*Pi*csgn(I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2-I*Pi*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1))^3-2*I*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d
*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d
*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+2*I*Pi*csg
n(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(
1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(
1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))+4*ln(2))*arcta
n(c*x)^2))+3*a*b^2*c^2*(-1/(c*e*x+c*d)/e*arctan(c*x)^2+2/e*(arctan(c*x)*e/(
c^2*d^2+e^2)*ln(c*e*x+c*d)-1/2*arctan(c*x)/(c^2*d^2+e^2)*e*ln(c^2*x^2+1)+1/
2/(c^2*d^2+e^2)*d*c*arctan(c*x)^2-e^2/(c^2*d^2+e^2)*(1/2*I*ln(c*e*x+c*d)*(-
ln((I*e-c*e*x)/(c*d+I*e))+ln((I*e+c*e*x)/(I*e-c*d)))/e-1/2*I*(dilog((I*e-c*
e*x)/(c*d+I*e))-dilog((I*e+c*e*x)/(I*e-c*d)))/e)+1/2*e/(c^2*d^2+e^2)*(-1/2*
I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I
))-1/2*ln(c*x-I)^2)+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(
c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2))))+3*a^2*b*c^2*(-1/(c*e*x+c*d)/e*
arctan(c*x)+1/e*(e/(c^2*d^2+e^2)*ln(c*e*x+c*d)+1/(c^2*d^2+e^2)*(-1/2*e*ln(c
^2*x^2+1)+d*c*arctan(c*x)))))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^3}{(d+ex)^2}\,dx$$

```
[In] integrate((a+b*atan(c*x))**3/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 3/2*((2*c*d*arctan(c*x)/(c^2*d^2*e + e^3) - log(c^2*x^2 + 1)/(c^2*d^2 + e^2
) + 2*log(e*x + d)/(c^2*d^2 + e^2))*c - 2*arctan(c*x)/(e^2*x + d*e))*a^2*b
- a^3/(e^2*x + d*e) - 1/32*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^2
*x^2 + 1)^2 - 32*(e^2*x + d*e)*integrate(1/32*(28*(b^3*c^2*e*x^2 + b^3*e)*a
rctan(c*x)^3 + 12*(8*a*b^2*c^2*e*x^2 + b^3*c*e*x + b^3*c*d + 8*a*b^2*e)*arc
tan(c*x)^2 - 12*(b^3*c^2*e*x^2 + b^3*c^2*d*x)*arctan(c*x)*log(c^2*x^2 + 1)
- 3*(b^3*c*e*x + b^3*c*d - (b^3*c^2*e*x^2 + b^3*e)*arctan(c*x))*log(c^2*x^2
 + 1)^2)/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e +
e^3)*x^2), x))/(e^2*x + d*e)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+e\,x)^2}\,dx$$

```
[In] int((a + b*atan(c*x))^3/(d + e*x)^2,x)
[Out] int((a + b*atan(c*x))^3/(d + e*x)^2, x)
```

3.20 $\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 936

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx = \frac{3bc^3d(a+b\arctan(cx))^2}{2(c^2d^2+e^2)^2} + \frac{3ibc^2e(a+b\arctan(cx))^2}{2(c^2d^2+e^2)^2} - \frac{3bc(a+b\arctan(cx))^2}{2(c^2d^2+e^2)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^3}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^3}{2e(c^2d^2+e^2)^2} - \frac{(a+b\arctan(cx))^3}{2e(d+ex)^2} - \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} - \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} + \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} + \frac{3ib^3c^2e\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)^2} + \frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{3ib^3c^2e\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2(c^2d^2+e^2)^2} + \frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} - \frac{3ib^3c^2e\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2} - \frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} - \frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)^2} + \frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2(c^2d^2+e^2)^2} + \frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2}$$

```
[Out] 3/2*b*c^3*d*(a+b*arctan(c*x))^2/(c^2*d^2+e^2)^2+3/2*I*b^3*c^2*e*polylog(2,1
-2/(1-I*c*x))/(c^2*d^2+e^2)^2-3/2*b*c*(a+b*arctan(c*x))^2/(c^2*d^2+e^2)/(e*
x+d)+3*I*b^2*c^3*d*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/(c^2*d^2+e^2)
^2+1/2*c^2*(c*d-e)*(c*d+e)*(a+b*arctan(c*x))^3/e/(c^2*d^2+e^2)^2-1/2*(a+b*a
rctan(c*x))^3/e/(e*x+d)^2-3*b^2*c^2*e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/(c^
2*d^2+e^2)^2-3*b*c^3*d*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/(c^2*d^2+e^2)^2+
3*b^2*c^2*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/(c^2*d^2+e^2)^2+3*b*c^3*d*(a+
b*arctan(c*x))^2*ln(2/(1+I*c*x))/(c^2*d^2+e^2)^2+3*b^2*c^2*e*(a+b*arctan(c*
x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d^2+e^2)^2+3*b*c^3*d*(a+b*arct
an(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d^2+e^2)^2+3/2*I*b^3*c^
2*e*polylog(2,1-2/(1+I*c*x))/(c^2*d^2+e^2)^2+I*c^3*d*(a+b*arctan(c*x))^3/(c
^2*d^2+e^2)^2-3/2*I*b^3*c^2*e*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/
(c^2*d^2+e^2)^2-3*I*b^2*c^3*d*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*
d+I*e)/(1-I*c*x))/(c^2*d^2+e^2)^2+3/2*I*b*c^2*e*(a+b*arctan(c*x))^2/(c^2*d^
2+e^2)^2+3*I*b^2*c^3*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/(c^2*d^2+
e^2)^2-3/2*b^3*c^3*d*polylog(3,1-2/(1-I*c*x))/(c^2*d^2+e^2)^2+3/2*b^3*c^3*d
*polylog(3,1-2/(1+I*c*x))/(c^2*d^2+e^2)^2+3/2*b^3*c^3*d*polylog(3,1-2*c*(e*
x+d)/(c*d+I*e)/(1-I*c*x))/(c^2*d^2+e^2)^2
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 936, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {4974, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964, 4968, 5114, 6745}

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx = \frac{3ic^2e\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^3}{2\left(c^2d^2+e^2\right)^2}+\frac{3ic^2e\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^3}{2\left(c^2d^2+e^2\right)^2}-\frac{3ic^2e\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)b^3}{2\left(c^2d^2+e^2\right)^2}-\frac{3c^3d\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^3}{2\left(c^2d^2+e^2\right)^2}+\frac{3c^3d\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)b^3}{2\left(c^2d^2+e^2\right)^2}+\frac{3c^3d\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)b^3}{2\left(c^2d^2+e^2\right)^2}-\frac{3c^2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)b^2}{\left(c^2d^2+e^2\right)^2}+\frac{3c^2e(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)b^2}{\left(c^2d^2+e^2\right)^2}+\frac{3c^2e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)b^2}{\left(c^2d^2+e^2\right)^2}+\frac{3ic^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{\left(c^2d^2+e^2\right)^2}+\frac{3ic^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{\left(c^2d^2+e^2\right)^2}-\frac{3ic^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)b^2}{\left(c^2d^2+e^2\right)^2}-\frac{3c(a+b\arctan(cx))^2b}{2\left(c^2d^2+e^2\right)(d+ex)}+\frac{3c^3d(a+b\arctan(cx))^2b}{2\left(c^2d^2+e^2\right)^2}+\frac{3ic^2e(a+b\arctan(cx))^2b}{2\left(c^2d^2+e^2\right)^2}-\frac{3c^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)b}{\left(c^2d^2+e^2\right)^2}+\frac{3c^3d(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)b}{\left(c^2d^2+e^2\right)^2}+\frac{3c^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)b}{\left(c^2d^2+e^2\right)^2}+\frac{ic^3d(a+b\arctan(cx))^3}{\left(c^2d^2+e^2\right)^2}+\frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^3}{2e\left(c^2d^2+e^2\right)^2}-\frac{(a+b\arctan(cx))^3}{2e(d+ex)^2}$$

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + e*x)^3,x]
```

```
[Out] (3*b*c^3*d*(a + b*ArcTan[c*x])^2)/(2*(c^2*d^2 + e^2)^2) + (((3*I)/2)*b*c^2*
e*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2)^2 - (3*b*c*(a + b*ArcTan[c*x])^2)/
(2*(c^2*d^2 + e^2)*(d + e*x)) + (I*c^3*d*(a + b*ArcTan[c*x])^3)/(c^2*d^2 +
e^2)^2 + (c^2*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^3)/(2*e*(c^2*d^2 + e^
2)^2) - (a + b*ArcTan[c*x])^3/(2*e*(d + e*x)^2) - (3*b^2*c^2*e*(a + b*ArcTa
n[c*x])*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 - (3*b*c^3*d*(a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b^2*c^2*e*(a + b*ArcTan[c*
x])*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b*c^3*d*(a + b*ArcTan[c*x])^
2*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b^2*c^2*e*(a + b*ArcTan[c*x])*
Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2 + (3*b*c^
3*d*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(
c^2*d^2 + e^2)^2 + (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2
*d^2 + e^2)^2 + ((3*I)*b^2*c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
I*c*x)])/(c^2*d^2 + e^2)^2 + (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 - 2/(1 + I*c
*x)])/(c^2*d^2 + e^2)^2 + ((3*I)*b^2*c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 - (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2 - ((3*I)*b^2*
c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
I*c*x))])/(c^2*d^2 + e^2)^2 - (3*b^3*c^3*d*PolyLog[3, 1 - 2/(1 - I*c*x)])/(
2*(c^2*d^2 + e^2)^2) + (3*b^3*c^3*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*(c^2*
d^2 + e^2)^2) + (3*b^3*c^3*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
 - I*c*x))])/(2*(c^2*d^2 + e^2)^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

integral

$$= -\frac{(a+b\arctan(cx))^3}{2e(d+ex)^2} + \frac{(3bc)\int\left(\frac{e^2(a+b\arctan(cx))^2}{(c^2d^2+e^2)(d+ex)^2} + \frac{2c^2de^2(a+b\arctan(cx))^2}{(c^2d^2+e^2)^2(d+ex)} + \frac{(c^4d^2-c^2e^2-2c^4dex)(a+b\arctan(cx))^2}{(c^2d^2+e^2)^2(1+c^2x^2)}\right)dx}{2e}$$

$$= -\frac{(a+b\arctan(cx))^3}{2e(d+ex)^2} + \frac{(3bc)\int\frac{(c^4d^2-c^2e^2-2c^4dex)(a+b\arctan(cx))^2}{1+c^2x^2}dx}{2e\left(c^2d^2+e^2\right)^2} + \frac{(3bc^3de)\int\frac{(a+b\arctan(cx))^2}{d+ex}dx}{\left(c^2d^2+e^2\right)^2} + \frac{(3bce)\int\frac{(a+b\arctan(cx))^2}{(d+ex)^2}dx}{2\left(c^2d^2+e^2\right)}$$

$$= -\frac{3bc(a+b\arctan(cx))^2}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{(a+b\arctan(cx))^3}{2e(d+ex)^2}$$
$$- \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} + \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2}$$
$$+ \frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2}$$
$$- \frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2}$$
$$- \frac{3b^3c^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)^2} + \frac{3b^3c^3d\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)^2}$$
$$+ \frac{(3bc)\int\left(\frac{c^4d^2\left(1-\frac{e^2}{c^2d^2}\right)(a+b\arctan(cx))^2}{1+c^2x^2} - \frac{2c^4dex(a+b\arctan(cx))^2}{1+c^2x^2}\right)dx}{2e\left(c^2d^2+e^2\right)^2}$$
$$+ \frac{\left(3b^2c^2\right)\int\left(\frac{e^2(a+b\arctan(cx))}{(c^2d^2+e^2)(d+ex)} + \frac{c^2(d-ex)(a+b\arctan(cx))}{(c^2d^2+e^2)(1+c^2x^2)}\right)dx}{c^2d^2+e^2}$$

$$= -\frac{3bc(a+b\arctan(cx))^2}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{(a+b\arctan(cx))^3}{2e(d+ex)^2}$$
$$- \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} + \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2}$$
$$+ \frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2}$$
$$- \frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2}$$
$$- \frac{3b^3c^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)^2} + \frac{3b^3c^3d\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)^2}$$
$$+ \frac{\left(3b^2c^4\right)\int\frac{(d-ex)(a+b\arctan(cx))}{1+c^2x^2}dx}{\left(c^2d^2+e^2\right)^2} - \frac{\left(3bc^5d\right)\int\frac{x(a+b\arctan(cx))^2}{1+c^2x^2}dx}{\left(c^2d^2+e^2\right)^2}$$
$$+ \frac{\left(3b^2c^2e^2\right)\int\frac{a+b\arctan(cx)}{d+ex}dx}{\left(c^2d^2+e^2\right)^2} + \frac{\left(3bc^3(cd-e)(cd+e)\right)\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}dx}{2e\left(c^2d^2+e^2\right)^2}$$

$$= -\frac{3bc(a+b\arctan(cx))^2}{2(c^2d^2+e^2)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^3}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^3}{2e(c^2d^2+e^2)^2}$$
$$- \frac{(a+b\arctan(cx))^3}{2e(d+ex)^2} - \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2}$$
$$- \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$
$$+ \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$
$$+ \frac{3ib^2c^3d(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2}$$
$$- \frac{3ib^2c^3d(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} - \frac{3b^3c^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)^2}$$
$$+ \frac{3b^3c^3d\,\mathrm{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2} + \frac{(3b^2c^4)\int\left(\frac{d(a+b\arctan(cx))}{1+c^2x^2} - \frac{ex(a+b\arctan(cx))}{1+c^2x^2}\right)dx}{(c^2d^2+e^2)^2}$$
$$+ \frac{(3bc^4d)\int\frac{(a+b\arctan(cx))^2}{i-cx}dx}{(c^2d^2+e^2)^2} + \frac{(3b^3c^3e)\int\frac{\log\left(\frac{2}{1-icx}\right)}{1+c^2x^2}dx}{(c^2d^2+e^2)^2} - \frac{(3b^3c^3e)\int\frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{1+c^2x^2}dx}{(c^2d^2+e^2)^2}$$

$$= -\frac{3bc(a+b\arctan(cx))^2}{2(c^2d^2+e^2)(d+ex)} + \frac{ic^3d(a+b\arctan(cx))^3}{(c^2d^2+e^2)^2}$$
$$+ \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^3}{2e(c^2d^2+e^2)^2} - \frac{(a+b\arctan(cx))^3}{2e(d+ex)^2}$$
$$- \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} - \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2}$$
$$+ \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$
$$+ \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$
$$+ \frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} - \frac{3ib^3c^2e\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2}$$
$$- \frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$
$$- \frac{3b^3c^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)^2} + \frac{3b^3c^3d\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2}$$
$$+ \frac{(3b^2c^4d)\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2} - \frac{(6b^2c^4d)\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2}$$
$$+ \frac{(3ib^3c^2e)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-icx}\right)}{(c^2d^2+e^2)^2} - \frac{(3b^2c^4e)\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2}$$

$$
\begin{aligned}
&= \frac{3bc^3d(a+b\arctan(cx))^2}{2\left(c^2d^2+e^2\right)^2}+\frac{3ibc^2e(a+b\arctan(cx))^2}{2\left(c^2d^2+e^2\right)^2}-\frac{3bc(a+b\arctan(cx))^2}{2\left(c^2d^2+e^2\right)(d+ex)} \\
&+\frac{ic^3d(a+b\arctan(cx))^3}{\left(c^2d^2+e^2\right)^2}+\frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^3}{2e\left(c^2d^2+e^2\right)^2}-\frac{(a+b\arctan(cx))^3}{2e(d+ex)^2} \\
&-\frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2}-\frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} \\
&+\frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2} \\
&+\frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{3ib^3c^2e\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)^2} \\
&+\frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} \\
&+\frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{\left(c^2d^2+e^2\right)^2}-\frac{3ib^3c^2e\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)^2} \\
&-\frac{3ib^2c^3d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2} \\
&-\frac{3b^3c^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)^2}+\frac{3b^3c^3d\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)^2} \\
&-\frac{\left(3ib^3c^4d\right)\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{\left(c^2d^2+e^2\right)^2}+\frac{\left(3b^2c^3e\right)\int\frac{a+b\arctan(cx)}{i-cx}\,dx}{\left(c^2d^2+e^2\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3bc^3d(a+b\arctan(cx))^2}{2(c^2d^2+e^2)^2} + \frac{3ibc^2e(a+b\arctan(cx))^2}{2(c^2d^2+e^2)^2} - \frac{3bc(a+b\arctan(cx))^2}{2(c^2d^2+e^2)(d+ex)} \\
&+ \frac{ic^3d(a+b\arctan(cx))^3}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^3}{2e(c^2d^2+e^2)^2} \\
&- \frac{(a+b\arctan(cx))^3}{2e(d+ex)^2} - \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} \\
&- \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} \\
&+ \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} \\
&+ \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} + \frac{3ib^3c^2e\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)^2} \\
&+ \frac{3ib^2c^3d(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} \\
&+ \frac{3ib^2c^3d(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} - \frac{3ib^3c^2e\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2} \\
&- \frac{3ib^2c^3d(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} \\
&- \frac{3b^3c^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)^2} + \frac{3b^3c^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2(c^2d^2+e^2)^2} \\
&+ \frac{3b^3c^3d\,\mathrm{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2} - \frac{(3b^3c^3e)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{(c^2d^2+e^2)^2}
\end{aligned}
$$

$$= \frac{3bc^3d(a+b\arctan(cx))^2}{2(c^2d^2+e^2)^2} + \frac{3ibc^2e(a+b\arctan(cx))^2}{2(c^2d^2+e^2)^2} - \frac{3bc(a+b\arctan(cx))^2}{2(c^2d^2+e^2)(d+ex)}$$
$$+ \frac{ic^3d(a+b\arctan(cx))^3}{(c^2d^2+e^2)^2} + \frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^3}{2e(c^2d^2+e^2)^2}$$
$$- \frac{(a+b\arctan(cx))^3}{2e(d+ex)^2} - \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2}$$
$$- \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2} + \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2}$$
$$+ \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} + \frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$
$$+ \frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2} + \frac{3ib^3c^2e\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)^2}$$
$$+ \frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{(c^2d^2+e^2)^2}$$
$$+ \frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{(c^2d^2+e^2)^2} - \frac{3ib^3c^2e\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2}$$
$$- \frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{(c^2d^2+e^2)^2}$$
$$- \frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2(c^2d^2+e^2)^2} + \frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2(c^2d^2+e^2)^2}$$
$$+ \frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2(c^2d^2+e^2)^2} + \frac{(3ib^3c^2e)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+icx}\right)}{(c^2d^2+e^2)^2}$$

$$
\begin{aligned}
&= \frac{3bc^3d(a+b\arctan(cx))^2}{2\left(c^2d^2+e^2\right)^2}+\frac{3ibc^2e(a+b\arctan(cx))^2}{2\left(c^2d^2+e^2\right)^2}-\frac{3bc(a+b\arctan(cx))^2}{2\left(c^2d^2+e^2\right)(d+ex)} \\
&\quad+\frac{ic^3d(a+b\arctan(cx))^3}{\left(c^2d^2+e^2\right)^2}+\frac{c^2(cd-e)(cd+e)(a+b\arctan(cx))^3}{2e\left(c^2d^2+e^2\right)^2} \\
&\quad-\frac{(a+b\arctan(cx))^3}{2e(d+ex)^2}-\frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} \\
&\quad-\frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{\left(c^2d^2+e^2\right)^2} \\
&\quad+\frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{3b^2c^2e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2} \\
&\quad+\frac{3bc^3d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{3ib^3c^2e\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)^2} \\
&\quad+\frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{3ib^3c^2e\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2\left(c^2d^2+e^2\right)^2} \\
&\quad+\frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{\left(c^2d^2+e^2\right)^2}-\frac{3ib^3c^2e\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)^2} \\
&\quad-\frac{3ib^2c^3d(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{\left(c^2d^2+e^2\right)^2}-\frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2\left(c^2d^2+e^2\right)^2} \\
&\quad+\frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2\left(c^2d^2+e^2\right)^2}+\frac{3b^3c^3d\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2\left(c^2d^2+e^2\right)^2}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx=\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx
$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^3,x]
[Out] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^3, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 93.31 (sec) , antiderivative size = 40258, normalized size of antiderivative = 43.01

method	result	size
derivativedivides	Expression too large to display	40258
default	Expression too large to display	40258
parts	Expression too large to display	40263

```
[In] int((a+b*arctan(c*x))^3/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx=\int \frac{(b\arctan(cx)+a)^3}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*atan(c*x))**3/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+ex)^3}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+e\,x)^3}\,dx$$

```
[In] int((a + b*atan(c*x))^3/(d + e*x)^3,x)
[Out] int((a + b*atan(c*x))^3/(d + e*x)^3, x)
```

3.21 $\int (d+ex)^2 \left(a+b\arctan\left(cx^2\right)\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 250

$$\begin{aligned}\int (d+ex)^2 \left(a+b\arctan\left(cx^2\right)\right) \, dx = & -\frac{2be^2x}{3c} - \frac{bd^3\arctan\left(cx^2\right)}{3e} \\ & + \frac{(d+ex)^3\left(a+b\arctan\left(cx^2\right)\right)}{3e} \\ & + \frac{b(3cd^2-e^2)\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} \\ & - \frac{b(3cd^2-e^2)\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} \\ & - \frac{b(3cd^2+e^2)\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}} \\ & + \frac{b(3cd^2+e^2)\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}} \\ & - \frac{bde\log\left(1+c^2x^4\right)}{2c}\end{aligned}$$

```
[Out] -2/3*b*e^2*x/c-1/3*b*d^3*arctan(c*x^2)/e+1/3*(e*x+d)^3*(a+b*arctan(c*x^2))/
e-1/2*b*d*e*ln(c^2*x^4+1)/c-1/6*b*(3*c*d^2-e^2)*arctan(-1+x*2^(1/2)*c^(1/2)
)/c^(3/2)*2^(1/2)-1/6*b*(3*c*d^2-e^2)*arctan(1+x*2^(1/2)*c^(1/2))/c^(3/2)*2
^(1/2)-1/12*b*(3*c*d^2+e^2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)+1
/12*b*(3*c*d^2+e^2)*ln(1+c*x^2+x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {4980, 1845, 1262, 649, 209, 266, 1294, 1182, 1176, 631, 210, 1179, 642}

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{(d+ex)^3\left(a+b\arctan\left(cx^2\right)\right)}{3e} + \frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)\left(3cd^2-e^2\right)}{3\sqrt{2}c^{3/2}} - \frac{b\arctan\left(\sqrt{2}\sqrt{c}x+1\right)\left(3cd^2-e^2\right)}{3\sqrt{2}c^{3/2}} - \frac{bd^3\arctan\left(cx^2\right)}{3e} - \frac{b(3cd^2+e^2)\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}} + \frac{b(3cd^2+e^2)\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}} - \frac{bde\log\left(c^2x^4+1\right)}{2c} - \frac{2be^2x}{3c}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*e^2*x)/(3*c) - (b*d^3*ArcTan[c*x^2])/(3*e) + ((d + e*x)^3*(a + b*ArcT
an[c*x^2]))/(3*e) + (b*(3*c*d^2 - e^2)*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(3*Sq
rt[2]*c^(3/2)) - (b*(3*c*d^2 - e^2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*Sqrt[
2]*c^(3/2)) - (b*(3*c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqr
t[2]*c^(3/2)) + (b*(3*c*d^2 + e^2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*S
qrt[2]*c^(3/2)) - (b*d*e*Log[1 + c^2*x^4])/(2*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_) + (e_.)*(x_))^(m_.), x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(e*(m + 1))), x] - D
ist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 + c^2*x^(2*n))
), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^3\,(a+b\arctan(cx^2))}{3e} - \frac{(2bc)\int \frac{x(d+ex)^3}{1+c^2x^4}\,dx}{3e}\\
&= \frac{(d+ex)^3\,(a+b\arctan(cx^2))}{3e} - \frac{(2bc)\int \left(\frac{x(d^3+3de^2x^2)}{1+c^2x^4} + \frac{x^2(3d^2e+e^3x^2)}{1+c^2x^4}\right)dx}{3e}\\
&= \frac{(d+ex)^3\,(a+b\arctan(cx^2))}{3e} - \frac{(2bc)\int \frac{x(d^3+3de^2x^2)}{1+c^2x^4}\,dx}{3e} - \frac{(2bc)\int \frac{x^2(3d^2e+e^3x^2)}{1+c^2x^4}\,dx}{3e}\\
&= -\frac{2be^2x}{3c} + \frac{(d+ex)^3\,(a+b\arctan(cx^2))}{3e}\\
&\quad + \frac{(2b)\int \frac{e^3-3c^2d^2ex^2}{1+c^2x^4}\,dx}{3ce} - \frac{(bc)\mathrm{Subst}\left(\int \frac{d^3+3de^2x}{1+c^2x^2}\,dx, x, x^2\right)}{3e}\\
&= -\frac{2be^2x}{3c} + \frac{(d+ex)^3\,(a+b\arctan(cx^2))}{3e}\\
&\quad - \frac{(bcd^3)\,\mathrm{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, x^2\right)}{3e} - (bcde)\mathrm{Subst}\left(\int \frac{x}{1+c^2x^2}\,dx, x, x^2\right)\\
&\quad - \frac{(b(3cd^2-e^2))\int \frac{c+c^2x^2}{1+c^2x^4}\,dx}{3c^2} + \frac{(b(3cd^2+e^2))\int \frac{c-c^2x^2}{1+c^2x^4}\,dx}{3c^2}
\end{aligned}
$$

$$= -\frac{2be^2x}{3c} - \frac{bd^3\arctan(cx^2)}{3e} + \frac{(d+ex)^3(a+b\arctan(cx^2))}{3e} - \frac{bde\log(1+c^2x^4)}{2c} - \frac{(b(3cd^2-e^2))\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}dx}{6c^2} - \frac{(b(3cd^2-e^2))\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}dx}{6c^2} - \frac{(b(3cd^2+e^2))\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}dx}{6\sqrt{2}c^{3/2}} - \frac{(b(3cd^2+e^2))\int\frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}dx}{6\sqrt{2}c^{3/2}}$$

$$= -\frac{2be^2x}{3c} - \frac{bd^3\arctan(cx^2)}{3e} + \frac{(d+ex)^3(a+b\arctan(cx^2))}{3e} - \frac{b(3cd^2+e^2)\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}} + \frac{b(3cd^2+e^2)\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}} - \frac{bde\log(1+c^2x^4)}{2c} - \frac{(b(3cd^2-e^2))\operatorname{Subst}\left(\int\frac{1}{-1-x^2}dx, x, 1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} + \frac{(b(3cd^2-e^2))\operatorname{Subst}\left(\int\frac{1}{-1-x^2}dx, x, 1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}$$

$$= -\frac{2be^2x}{3c} - \frac{bd^3\arctan(cx^2)}{3e} + \frac{(d+ex)^3(a+b\arctan(cx^2))}{3e} + \frac{b(3cd^2-e^2)\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} - \frac{b(3cd^2-e^2)\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} - \frac{b(3cd^2+e^2)\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}} + \frac{b(3cd^2+e^2)\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}} - \frac{bde\log(1+c^2x^4)}{2c}$$

Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.01

$$\int(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{1}{12}\left(12ad^2x - \frac{8be^2x}{c} + 12adex^2 + 4ae^2x^3 + 4bx\left(3d^2+3dex+e^2x^2\right)\arctan\left(cx^2\right) + \frac{2\sqrt{2}b(3cd^2-e^2)\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{c^{3/2}} - \frac{2\sqrt{2}b(3cd^2-e^2)\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{c^{3/2}} - \frac{\sqrt{2}b(3cd^2+e^2)\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{c^{3/2}} + \frac{\sqrt{2}b(3cd^2+e^2)\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{c^{3/2}} - \frac{6bde\log(1+c^2x^4)}{c}\right)$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (12*a*d^2*x - (8*b*e^2*x)/c + 12*a*d*e*x^2 + 4*a*e^2*x^3 + 4*b*x*(3*d^2 + 3
*d*e*x + e^2*x^2)*ArcTan[c*x^2] + (2*Sqrt[2]*b*(3*c*d^2 - e^2)*ArcTan[1 - S
qrt[2]*Sqrt[c]*x])/c^(3/2) - (2*Sqrt[2]*b*(3*c*d^2 - e^2)*ArcTan[1 + Sqrt[2
]*Sqrt[c]*x])/c^(3/2) - (Sqrt[2]*b*(3*c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c]*
x + c*x^2])/c^(3/2) + (Sqrt[2]*b*(3*c*d^2 + e^2)*Log[1 + Sqrt[2]*Sqrt[c]*x
+ c*x^2])/c^(3/2) - (6*b*d*e*Log[1 + c^2*x^4])/c)/12
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.21

method	result
default	$\frac{a(ex+d)^3}{3e}+b\left(\frac{e^2\arctan(c\,x^2)x^3}{3}+e\arctan(c\,x^2)\,d\,x^2+\arctan(c\,x^2)\,d^2x+\frac{\arctan(c\,x^2)d^3}{3e}-\frac{2c\left(\frac{e^3x}{c^2}+\frac{-\frac{e^3\left(\frac{1}{c}\right.}{}}{}\right.}{}\right.$
parts	$\frac{a(ex+d)^3}{3e}+b\left(\frac{e^2\arctan(c\,x^2)x^3}{3}+e\arctan(c\,x^2)\,d\,x^2+\arctan(c\,x^2)\,d^2x+\frac{\arctan(c\,x^2)d^3}{3e}-\frac{2c\left(\frac{e^3x}{c^2}+\frac{-\frac{e^3\left(\frac{1}{c}\right.}{}}{}\right.}{}\right.$

```
[In] int((e*x+d)^2*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(e*x+d)^3/e+b*(1/3*e^2*arctan(c*x^2)*x^3+e*arctan(c*x^2)*d*x^2+arctan
(c*x^2)*d^2*x+1/3/e*arctan(c*x^2)*d^3-2/3*c/e*(1/c^2*e^3*x+1/c^2*(-1/8*e^3*
(1/c^2)^(1/4)*2^(1/2)*(ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-
(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)
+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1))+1/2*c^2*d^3/(c^2)^(1/2)*arctan(x^2*(c
^2)^(1/2))+3/8*d^2*e/(1/c^2)^(1/4)*2^(1/2)*(ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)
+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/
2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1))+3/4*d*e^2*ln(c^2
*x^4+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. 2(201) = 402.

Time = 0.28 (sec) , antiderivative size = 1013, normalized size of antiderivative = 4.05

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^2\right)\right)\, dx$$

$$= \frac{2\,ace^2x^3+6\,acdex^2+2\,(3\,acd^2-2\,be^2)x+2\,(bce^2x^3+3\,bcdex^2+3\,bcd^2x)\arctan\left(cx^2\right)-\left(3\,bde+c\sqrt{\frac{6b}{}}\right.}{}$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c*e^2*x^3 + 6*a*c*d*e*x^2 + 2*(3*a*c*d^2 - 2*b*e^2)*x + 2*(b*c*e^2
*x^3 + 3*b*c*d*e*x^2 + 3*b*c*d^2*x)*arctan(c*x^2) - (3*b*d*e + c*sqrt((6*b^
2*d^2*e^2 + c^2*sqrt(-(81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))
/c^2))*log(-(81*b^3*c^4*d^8 - b^3*e^8)*x + (9*b^2*c^3*d^4*e^2 - b^2*c*e^6 -
 3*c^5*d^2*sqrt(-(81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt
((6*b^2*d^2*e^2 + c^2*sqrt(-(81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)
/c^6))/c^2)) - (3*b*d*e - c*sqrt((6*b^2*d^2*e^2 + c^2*sqrt(-(81*b^4*c^4*d^8
 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/c^2))*log(-(81*b^3*c^4*d^8 - b^3*e^8
)*x - (9*b^2*c^3*d^4*e^2 - b^2*c*e^6 - 3*c^5*d^2*sqrt(-(81*b^4*c^4*d^8 - 18
*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt((6*b^2*d^2*e^2 + c^2*sqrt(-(81*b^4*c
^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/c^2)) - (3*b*d*e + c*sqrt((6*b
^2*d^2*e^2 - c^2*sqrt(-(81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)
)/c^2))*log(-(81*b^3*c^4*d^8 - b^3*e^8)*x + (9*b^2*c^3*d^4*e^2 - b^2*c*e^6
+ 3*c^5*d^2*sqrt(-(81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqr
t((6*b^2*d^2*e^2 - c^2*sqrt(-(81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8
)/c^6))/c^2)) - (3*b*d*e - c*sqrt((6*b^2*d^2*e^2 - c^2*sqrt(-(81*b^4*c^4*d^
8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/c^2))*log(-(81*b^3*c^4*d^8 - b^3*e^
8)*x - (9*b^2*c^3*d^4*e^2 - b^2*c*e^6 + 3*c^5*d^2*sqrt(-(81*b^4*c^4*d^8 - 1
8*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt((6*b^2*d^2*e^2 - c^2*sqrt(-(81*b^4*
c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/c^2)))/c
```

Sympy [A] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.61

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$= \begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\,\mathrm{atan}\left(cx^2\right) + bdex^2\,\mathrm{atan}\left(cx^2\right) + \frac{be^2x^3\,\mathrm{atan}\left(cx^2\right)}{3} - \frac{bd^2\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{c\sqrt[4]{-\frac{1}{c^2}}} + \frac{bd^2\log\left(x\right.}{2c\sqrt[4]{}} \\ a\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*atan(c*x**2) +
b*d*e*x**2*atan(c*x**2) + b*e**2*x**3*atan(c*x**2)/3 - b*d**2*log(x - (-1/c
**2)**(1/4))/(c*(-1/c**2)**(1/4)) + b*d**2*log(x**2 + sqrt(-1/c**2))/(2*c*(
-1/c**2)**(1/4)) - b*d**2*atan(x/(-1/c**2)**(1/4))/(c*(-1/c**2)**(1/4)) - b
*d*e*log(x**2 + sqrt(-1/c**2))/c - 2*b*e**2*x/(3*c) - b*d**2*atan(c*x**2)/(
c**2*(-1/c**2)**(3/4)) + b*d*e*atan(c*x**2)/(c**2*sqrt(-1/c**2)) - b*e**2*a
tan(c*x**2)/(3*c**2*(-1/c**2)**(1/4)) + b*e**2*log(x - (-1/c**2)**(1/4))/(3
*c**3*(-1/c**2)**(3/4)) - b*e**2*log(x**2 + sqrt(-1/c**2))/(6*c**3*(-1/c**2
)**(3/4)) - b*e**2*atan(x/(-1/c**2)**(1/4))/(3*c**3*(-1/c**2)**(3/4)), Ne(c
, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.29

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{1}{3}\,ae^2x^3 + adex^2$$
$$- \frac{1}{4}\left(\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}} + \right.\right.\right.$$
$$+ \frac{1}{12}\left(4\,x^3\arctan\left(cx^2\right) - c\left(\frac{8\,x}{c^2} - \frac{-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right.}{\sqrt{c}}}{c^2}\right.\right.$$
$$+ ad^2x + \frac{\left(2\,cx^2\arctan\left(cx^2\right) - \log\left(c^2x^4+1\right)\right)bde}{2\,c}$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 - 1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + s
qrt(2)*sqrt(c))/sqrt(c))/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sq
rt(2)*sqrt(c))/sqrt(c))/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1
)/c^(3/2) + sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arcta
n(c*x^2))*b*d^2 + 1/12*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2
 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x +
 1)/sqrt(c))/c^2))*b*e^2 + a*d^2*x + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x
^4 + 1))*b*d*e/c
```

Giac [A] (verification not implemented)

none

Time = 1.05 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.22

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^2\right)\right)\, dx = -\frac{bde\log\left(c^2x^4+1\right)}{2\,c} + \frac{bce^2x^3\arctan\left(cx^2\right)+ace^2x^3+3\,bcdex^2\arctan\left(cx^2\right)+3\,acdex^2+3\,bcd^2x\arctan\left(cx^2\right)+3\,acd^2x-2\,be}{3\,c} - \frac{\sqrt{2}(3\,bc^2d^2-be^2|c|)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{6\,c|c|^{\frac{3}{2}}} - \frac{\sqrt{2}(3\,bc^2d^2-be^2|c|)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{6\,c|c|^{\frac{3}{2}}} + \frac{\sqrt{2}(3\,bc^2d^2+be^2|c|)\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{12\,c|c|^{\frac{3}{2}}} - \frac{\sqrt{2}\left(3\,bc^2d^2\sqrt{|c|}+be^2|c|^{\frac{3}{2}}\right)\log\left(x^2-\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{12\,c^3}$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] -1/2*b*d*e*log(c^2*x^4 + 1)/c + 1/3*(b*c*e^2*x^3*arctan(c*x^2) + a*c*e^2*x^
3 + 3*b*c*d*e*x^2*arctan(c*x^2) + 3*a*c*d*e*x^2 + 3*b*c*d^2*x*arctan(c*x^2)
 + 3*a*c*d^2*x - 2*b*e^2*x)/c - 1/6*sqrt(2)*(3*b*c^2*d^2 - b*e^2*abs(c))*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/(c*abs(c)^(3/2)
) - 1/6*sqrt(2)*(3*b*c^2*d^2 - b*e^2*abs(c))*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)/sqrt(abs(c)))*sqrt(abs(c)))/(c*abs(c)^(3/2)) + 1/12*sqrt(2)*(3*b*c^2*d^
2 + b*e^2*abs(c))*log(x^2 + sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/(c*abs(c)^(3
/2)) - 1/12*sqrt(2)*(3*b*c^2*d^2*sqrt(abs(c)) + b*e^2*abs(c)^(3/2))*log(x^2
 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^3
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.68

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{a\,e^2\,x^3}{3}+a\,d^2\,x+\frac{b\,e^2\,x^3\,\mathrm{atan}\left(c\,x^2\right)}{3}+a\,d\,e\,x^2-\frac{3\,b\,d^2\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}-1\right)\,\sqrt{\frac{1\mathrm{i}}{9\,c}}}{2}+\frac{3\,b\,d^2\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}+1\right)\,\sqrt{\frac{1\mathrm{i}}{9\,c}}}{2}-\frac{b\,d^2\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}+1\mathrm{i}\right)\,\sqrt{\frac{1\mathrm{i}}{9\,c}}\,3\mathrm{i}}{2}+\frac{b\,d^2\,\ln\left(1+c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}\,3\mathrm{i}\right)\,\sqrt{\frac{1\mathrm{i}}{9\,c}}\,3\mathrm{i}}{2}-\frac{2\,b\,e^2\,x}{3\,c}+b\,d^2\,x\,\mathrm{atan}\left(c\,x^2\right)+\frac{b\,e^2\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}-1\right)\,\sqrt{\frac{1\mathrm{i}}{9\,c}}\,1\mathrm{i}}{2\,c}-\frac{b\,e^2\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}+1\right)\,\sqrt{\frac{1\mathrm{i}}{9\,c}}\,1\mathrm{i}}{2\,c}+\frac{b\,e^2\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}+1\mathrm{i}\right)\,\sqrt{\frac{1\mathrm{i}}{9\,c}}}{2\,c}-\frac{b\,e^2\,\ln\left(1+c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}\,3\mathrm{i}\right)\,\sqrt{\frac{1\mathrm{i}}{9\,c}}}{2\,c}+b\,d\,e\,x^2\,\mathrm{atan}\left(c\,x^2\right)-\frac{b\,d\,e\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}-1\right)}{2\,c}-\frac{b\,d\,e\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}+1\right)}{2\,c}-\frac{b\,d\,e\,\ln\left(3\,c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}+1\mathrm{i}\right)}{2\,c}-\frac{b\,d\,e\,\ln\left(1+c\,x\,\sqrt{\frac{1\mathrm{i}}{9\,c}}\,3\mathrm{i}\right)}{2\,c}$$

```
[In] int((a + b*atan(c*x^2))*(d + e*x)^2,x)
```

```
[Out] (a*e^2*x^3)/3 + a*d^2*x + (b*e^2*x^3*atan(c*x^2))/3 + a*d*e*x^2 - (3*b*d^2*
log(3*c*x*(1i/(9*c))^(1/2) - 1)*(1i/(9*c))^(1/2))/2 + (3*b*d^2*log(3*c*x*(1
i/(9*c))^(1/2) + 1)*(1i/(9*c))^(1/2))/2 - (b*d^2*log(3*c*x*(1i/(9*c))^(1/2)
 + 1i)*(1i/(9*c))^(1/2)*3i)/2 + (b*d^2*log(c*x*(1i/(9*c))^(1/2)*3i + 1)*(1i
/(9*c))^(1/2)*3i)/2 - (2*b*e^2*x)/(3*c) + b*d^2*x*atan(c*x^2) + (b*e^2*log(
3*c*x*(1i/(9*c))^(1/2) - 1)*(1i/(9*c))^(1/2)*1i)/(2*c) - (b*e^2*log(3*c*x*(
1i/(9*c))^(1/2) + 1)*(1i/(9*c))^(1/2)*1i)/(2*c) + (b*e^2*log(3*c*x*(1i/(9*c
```

```
))^(1/2) + 1i)*(1i/(9*c))^(1/2))/(2*c) - (b*e^2*log(c*x*(1i/(9*c))^(1/2)*3i
 + 1)*(1i/(9*c))^(1/2))/(2*c) + b*d*e*x^2*atan(c*x^2) - (b*d*e*log(3*c*x*(1
i/(9*c))^(1/2) - 1))/(2*c) - (b*d*e*log(3*c*x*(1i/(9*c))^(1/2) + 1))/(2*c)
- (b*d*e*log(3*c*x*(1i/(9*c))^(1/2) + 1i))/(2*c) - (b*d*e*log(c*x*(1i/(9*c)
)^(1/2)*3i + 1))/(2*c)
```

3.22 $\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 192

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{bd^2\arctan\left(cx^2\right)}{2e}+\frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e}+\frac{bd\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{bd\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{bd\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}+\frac{bd\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}-\frac{be\log\left(1+c^2x^4\right)}{4c}$$

```
[Out] -1/2*b*d^2*arctan(c*x^2)/e+1/2*(e*x+d)^2*(a+b*arctan(c*x^2))/e-1/4*b*e*ln(c
^2*x^4+1)/c-1/2*b*d*arctan(-1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/2*b*d*ar
ctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/4*b*d*ln(1+c*x^2-x*2^(1/2)*c^(1
/2))*2^(1/2)/c^(1/2)+1/4*b*d*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules

used = {4980, 1845, 303, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e} - \frac{bd^2\arctan\left(cx^2\right)}{2e} + \frac{bd\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} - \frac{bd\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}} - \frac{be\log\left(c^2x^4+1\right)}{4c} - \frac{bd\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}} + \frac{bd\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}$$

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -1/2*(b*d^2*ArcTan[c*x^2])/e + ((d + e*x)^2*(a + b*ArcTan[c*x^2]))/(2*e) +
(b*d*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*d*ArcTan[1 + Sqr
t[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*d*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2
])/(2*Sqrt[2]*Sqrt[c]) + (b*d*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2
]*Sqrt[c]) - (b*e*Log[1 + c^2*x^4])/(4*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_) + (e_.)*(x_))^(m_.), x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(e*(m + 1))), x] - D
ist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 + c^2*x^(2*n))
), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e}-\frac{(bc)\int \frac{x(d+ex)^2}{1+c^2x^4}\,dx}{e}\\
&= \frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e}-\frac{(bc)\int \left(\frac{2dex^2}{1+c^2x^4}+\frac{x\left(d^2+e^2x^2\right)}{1+c^2x^4}\right)dx}{e}\\
&= \frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e}-(2bcd)\int \frac{x^2}{1+c^2x^4}\,dx-\frac{(bc)\int \frac{x\left(d^2+e^2x^2\right)}{1+c^2x^4}\,dx}{e}\\
&= \frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e}+(bd)\int \frac{1-cx^2}{1+c^2x^4}\,dx\\
&\quad -(bd)\int \frac{1+cx^2}{1+c^2x^4}\,dx-\frac{(bc)\text{Subst}\left(\int \frac{d^2+e^2x}{1+c^2x^2}\,dx,x,x^2\right)}{2e}\\
&= \frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e}-\frac{(bd)\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{2c}\\
&\quad -\frac{(bd)\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{2c}-\frac{(bd)\int \frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}\sqrt{c}}-\frac{(bd)\int \frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}\sqrt{c}}\\
&\quad -\frac{\left(bcd^2\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{2e}-\frac{1}{2}(bce)\text{Subst}\left(\int \frac{x}{1+c^2x^2}\,dx,x,x^2\right)\\
&= -\frac{bd^2\arctan\left(cx^2\right)}{2e}+\frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e}\\
&\quad -\frac{bd\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}+\frac{bd\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}-\frac{be\log\left(1+c^2x^4\right)}{4c}\\
&\quad -\frac{(bd)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}+\frac{(bd)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\\
&= -\frac{bd^2\arctan\left(cx^2\right)}{2e}+\frac{(d+ex)^2\left(a+b\arctan\left(cx^2\right)\right)}{2e}+\frac{bd\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\\
&\quad -\frac{bd\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{bd\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}\\
&\quad +\frac{bd\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}-\frac{be\log\left(1+c^2x^4\right)}{4c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx = adx+\frac{1}{2}aex^2+bdx\arctan\left(cx^2\right)+\frac{1}{2}bex^2\arctan\left(cx^2\right) - \frac{bd\left(-2\arctan\left(1-\sqrt{2}\sqrt{c}x\right)+2\arctan\left(1+\sqrt{2}\sqrt{c}x\right)+\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)-\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)\right)}{2\sqrt{2}\sqrt{c}} - \frac{be\log\left(1+c^2x^4\right)}{4c}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + b*d*x*ArcTan[c*x^2] + (b*e*x^2*ArcTan[c*x^2])/2 - (b*
d*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c]*x] + 2*ArcTan[1 + Sqrt[2]*Sqrt[c]*x] + Log
[1 - Sqrt[2]*Sqrt[c]*x + c*x^2] - Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]))/(2*S
qrt[2]*Sqrt[c]) - (b*e*Log[1 + c^2*x^4])/(4*c)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.76

method	result
default	$a\left(\frac{1}{2}e\,x^2+dx\right)+b\left(\frac{\arctan\left(c\,x^2\right)x^2e}{2}+\arctan\left(c\,x^2\right)dx-c\left(\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right.\right.}{4c^2\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right.\right.$
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+b\left(\frac{\arctan\left(c\,x^2\right)x^2e}{2}+\arctan\left(c\,x^2\right)dx-c\left(\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right.\right.}{4c^2\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right.\right.$

```
[In] int((e*x+d)*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e*x^2+d*x)+b*(1/2*arctan(c*x^2)*x^2*e+arctan(c*x^2)*d*x-c*(1/4*d/c^2
/(1/c^2)^(1/4)*2^(1/2)*(ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2
+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1
)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1))+1/4*e/c^2*ln(c^2*x^4+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.43

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{2\,acex^2+4\,acdx+2\left(bcex^2+2\,bcdx\right)\arctan\left(cx^2\right)-\left(be-2\,c\sqrt{-\sqrt{-\frac{b^4d^4}{c^2}}}\right)\log\left(b^3d^3x+\sqrt{-\frac{b^4d^4}{c^2}}c\sqrt{-\sqrt{}}\right.}{}$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c*e*x^2 + 4*a*c*d*x + 2*(b*c*e*x^2 + 2*b*c*d*x)*arctan(c*x^2) - (b
*e - 2*c*sqrt(-sqrt(-b^4*d^4/c^2)))*log(b^3*d^3*x + sqrt(-b^4*d^4/c^2)*c*sq
rt(-sqrt(-b^4*d^4/c^2))) - (b*e + 2*c*sqrt(-sqrt(-b^4*d^4/c^2)))*log(b^3*d^
3*x - sqrt(-b^4*d^4/c^2)*c*sqrt(-sqrt(-b^4*d^4/c^2))) - (b*e + 2*(-b^4*d^4/
c^2)^(1/4)*c)*log(b^3*d^3*x + (-b^4*d^4/c^2)^(3/4)*c) - (b*e - 2*(-b^4*d^4/
c^2)^(1/4)*c)*log(b^3*d^3*x - (-b^4*d^4/c^2)^(3/4)*c))/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 1266, normalized size of antiderivative = 6.59

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*(d*x + e*x**2/2), Eq(c, 0)), ((a - oo*I*b)*(d*x + e*x**2/2), E
q(c, -I/x**2)), ((a + oo*I*b)*(d*x + e*x**2/2), Eq(c, I/x**2)), (2*a*c**5*d
*x**5*(-1/c**2)**(11/4)/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3*(-1/c**2)**
(11/4)) + a*c**5*e*x**6*(-1/c**2)**(11/4)/(2*c**5*x**4*(-1/c**2)**(11/4) +
2*c**3*(-1/c**2)**(11/4)) + 2*a*c**3*d*x*(-1/c**2)**(11/4)/(2*c**5*x**4*(-1
/c**2)**(11/4) + 2*c**3*(-1/c**2)**(11/4)) + a*c**3*e*x**2*(-1/c**2)**(11/4
)/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3*(-1/c**2)**(11/4)) + 2*b*c**5*d*x
**5*(-1/c**2)**(11/4)*atan(c*x**2)/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3*
(-1/c**2)**(11/4)) + b*c**5*e*x**6*(-1/c**2)**(11/4)*atan(c*x**2)/(2*c**5*x
**4*(-1/c**2)**(11/4) + 2*c**3*(-1/c**2)**(11/4)) - 2*b*c**4*d*x**4*(-1/c**
2)**(5/2)*log(x - (-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3
*(-1/c**2)**(11/4)) + b*c**4*d*x**4*(-1/c**2)**(5/2)*log(x**2 + sqrt(-1/c**
2))/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3*(-1/c**2)**(11/4)) - 2*b*c**4*d
*x**4*(-1/c**2)**(5/2)*atan(x/(-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(11
```

```
/4) + 2*c**3*(-1/c**2)**(11/4)) - b*c**4*e*x**4*(-1/c**2)**(11/4)*log(x**2
+ sqrt(-1/c**2))/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3*(-1/c**2)**(11/4))
 + 2*b*c**3*d*x*(-1/c**2)**(11/4)*atan(c*x**2)/(2*c**5*x**4*(-1/c**2)**(11/
4) + 2*c**3*(-1/c**2)**(11/4)) + b*c**3*e*x**2*(-1/c**2)**(11/4)*atan(c*x**
2)/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3*(-1/c**2)**(11/4)) - 2*b*c**2*d*
(-1/c**2)**(5/2)*log(x - (-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(11/4) +
 2*c**3*(-1/c**2)**(11/4)) + b*c**2*d*(-1/c**2)**(5/2)*log(x**2 + sqrt(-1/c
**2))/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3*(-1/c**2)**(11/4)) - 2*b*c**2
*d*(-1/c**2)**(5/2)*atan(x/(-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(11/4)
 + 2*c**3*(-1/c**2)**(11/4)) - b*c**2*e*(-1/c**2)**(11/4)*log(x**2 + sqrt(-
1/c**2))/(2*c**5*x**4*(-1/c**2)**(11/4) + 2*c**3*(-1/c**2)**(11/4)) - 2*b*d
*x**4*atan(c*x**2)/(2*c**6*x**4*(-1/c**2)**(11/4) + 2*c**4*(-1/c**2)**(11/4
)) - 2*b*d*atan(c*x**2)/(2*c**8*x**4*(-1/c**2)**(11/4) + 2*c**6*(-1/c**2)**
(11/4)) + b*e*x**4*(-1/c**2)**(1/4)*atan(c*x**2)/(2*c**6*x**4*(-1/c**2)**(1
1/4) + 2*c**4*(-1/c**2)**(11/4)) + b*e*(-1/c**2)**(1/4)*atan(c*x**2)/(2*c**
8*x**4*(-1/c**2)**(11/4) + 2*c**6*(-1/c**2)**(11/4)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{2}\,aex^2 -\frac{1}{4}\left(\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{}}{}\right.\right.\right. + adx+\frac{(2\,cx^2\arctan\left(cx^2\right)-\log\left(c^2x^4+1\right))be}{4\,c}$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 - 1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c)
)/sqrt(c))/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))
/sqrt(c))/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sq
rt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*b*d
+ a*d*x + 1/4*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b*e/c
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{2}bex^2\arctan\left(cx^2\right)+\frac{1}{2}aex^2+bdx\arctan\left(cx^2\right)+adx-\frac{\sqrt{2}bcd\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{2|c|^{\frac{3}{2}}}-\frac{\sqrt{2}bcd\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{2|c|^{\frac{3}{2}}}+\frac{\left(\sqrt{2}bcd\sqrt{|c|}-bce\right)\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{4c^2}-\frac{\left(\sqrt{2}bcd\sqrt{|c|}+bce\right)\log\left(x^2-\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{4c^2}$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/2*b*e*x^2*arctan(c*x^2) + 1/2*a*e*x^2 + b*d*x*arctan(c*x^2) + a*d*x - 1/2
*sqrt(2)*b*c*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(abs(c))
)/abs(c)^(3/2) - 1/2*sqrt(2)*b*c*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/abs(c)^(3/2) + 1/4*(sqrt(2)*b*c*d*sqrt(abs(c)) - b*c*
e)*log(x^2 + sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 - 1/4*(sqrt(2)*b*c*d*sq
rt(abs(c)) + b*c*e)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)\,dx=a\,d\,x+\frac{a\,e\,x^2}{2}+b\,d\,x\,\mathrm{atan}\left(c\,x^2\right)-\frac{b\,e\,\ln\left(x\,\sqrt{-c\,1\mathrm{i}}-1\right)}{4\,c}-\frac{b\,e\,\ln\left(x\,\sqrt{-c\,1\mathrm{i}}+1\right)}{4\,c}-\frac{b\,e\,\ln\left(x\,\sqrt{c\,1\mathrm{i}}-1\right)}{4\,c}-\frac{b\,e\,\ln\left(x\,\sqrt{c\,1\mathrm{i}}+1\right)}{4\,c}+\frac{b\,e\,x^2\,\mathrm{atan}\left(c\,x^2\right)}{2}-\frac{b\,d\,\ln\left(x\,\sqrt{-c\,1\mathrm{i}}-1\right)\,\sqrt{-c\,1\mathrm{i}}}{2\,c}+\frac{b\,d\,\ln\left(x\,\sqrt{-c\,1\mathrm{i}}+1\right)\,\sqrt{-c\,1\mathrm{i}}}{2\,c}-\frac{b\,d\,\ln\left(x\,\sqrt{c\,1\mathrm{i}}-1\right)\,\sqrt{c\,1\mathrm{i}}}{2\,c}+\frac{b\,d\,\ln\left(x\,\sqrt{c\,1\mathrm{i}}+1\right)\,\sqrt{c\,1\mathrm{i}}}{2\,c}$$

```
[In] int((a + b*atan(c*x^2))*(d + e*x),x)
```

```
[Out] a*d*x + (a*e*x^2)/2 + b*d*x*atan(c*x^2) - (b*e*log(x*(-c*1i)^(1/2) - 1))/(4
*c) - (b*e*log(x*(-c*1i)^(1/2) + 1))/(4*c) - (b*e*log(x*(c*1i)^(1/2) - 1))/
(4*c) - (b*e*log(x*(c*1i)^(1/2) + 1))/(4*c) + (b*e*x^2*atan(c*x^2))/2 - (b*
d*log(x*(-c*1i)^(1/2) - 1)*(-c*1i)^(1/2))/(2*c) + (b*d*log(x*(-c*1i)^(1/2)
+ 1)*(-c*1i)^(1/2))/(2*c) - (b*d*log(x*(c*1i)^(1/2) - 1)*(c*1i)^(1/2))/(2*c
) + (b*d*log(x*(c*1i)^(1/2) + 1)*(c*1i)^(1/2))/(2*c)
```

3.23 $\int \frac{a+b\arctan(cx^2)}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 501

$$
\begin{aligned}
\int \frac{a+b\arctan(cx^2)}{d+ex}\,dx =& \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} \\
&+ \frac{bc\log\left(\frac{e\left(1-\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&+ \frac{bc\log\left(-\frac{e\left(1+\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&- \frac{bc\log\left(\frac{e\left(1-\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&- \frac{bc\log\left(-\frac{e\left(1+\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&+ \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[4]{-c^2}(d+ex)}{\sqrt[4]{-c^2}d-e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-c^2}}(d+ex)}{\sqrt{-\sqrt{-c^2}}d-e}\right)}{2\sqrt{-c^2}e} \\
&+ \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[4]{-c^2}(d+ex)}{\sqrt[4]{-c^2}d+e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-c^2}}(d+ex)}{\sqrt{-\sqrt{-c^2}}d+e}\right)}{2\sqrt{-c^2}e}
\end{aligned}
$$

```
[Out] (a+b*arctan(c*x^2))*ln(e*x+d)/e+1/2*b*c*ln(e*(1-(-c^2)^(1/4)*x)/((-c^2)^(1/
4)*d+e))*ln(e*x+d)/e/(-c^2)^(1/2)+1/2*b*c*ln(-e*(1+(-c^2)^(1/4)*x)/((-c^2)^
(1/4)*d-e))*ln(e*x+d)/e/(-c^2)^(1/2)-1/2*b*c*ln(e*x+d)*ln(e*(1-x*(-(-c^2)^(
```

```
1/2))^(1/2))/(e+d*(-(-c^2)^(1/2))^(1/2)))/e/(-c^2)^(1/2)-1/2*b*c*ln(e*x+d)*
ln(-e*(1+x*(-(-c^2)^(1/2))^(1/2))/(-e+d*(-(-c^2)^(1/2))^(1/2)))/e/(-c^2)^(1
/2)+1/2*b*c*polylog(2,(-c^2)^(1/4)*(e*x+d)/((-c^2)^(1/4)*d-e))/e/(-c^2)^(1/
2)+1/2*b*c*polylog(2,(-c^2)^(1/4)*(e*x+d)/((-c^2)^(1/4)*d+e))/e/(-c^2)^(1/2
)-1/2*b*c*polylog(2,(e*x+d)*(-(-c^2)^(1/2))^(1/2)/(-e+d*(-(-c^2)^(1/2))^(1/
2)))/e/(-c^2)^(1/2)-1/2*b*c*polylog(2,(e*x+d)*(-(-c^2)^(1/2))^(1/2)/(e+d*(-
(-c^2)^(1/2))^(1/2)))/e/(-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4976, 281, 209, 2463, 266, 2441, 2440, 2438}

$$\int \frac{a+b\arctan(cx^2)}{d+ex}\,dx=\frac{\log(d+ex)\left(a+b\arctan(cx^2)\right)}{e}+\frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[4]{-c^2}(d+ex)}{\sqrt[4]{-c^2}d-e}\right)}{2\sqrt{-c^2}e}-\frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-c^2}}(d+ex)}{\sqrt{-\sqrt{-c^2}}d-e}\right)}{2\sqrt{-c^2}e}+\frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[4]{-c^2}(d+ex)}{\sqrt[4]{-c^2}d+e}\right)}{2\sqrt{-c^2}e}-\frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-c^2}}(d+ex)}{\sqrt{-\sqrt{-c^2}}d+e}\right)}{2\sqrt{-c^2}e}+\frac{bc\log(d+ex)\log\left(\frac{e\left(1-\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d+e}\right)}{2\sqrt{-c^2}e}+\frac{bc\log(d+ex)\log\left(-\frac{e\left(\sqrt[4]{-c^2}x+1\right)}{\sqrt[4]{-c^2}d-e}\right)}{2\sqrt{-c^2}e}-\frac{bc\log(d+ex)\log\left(\frac{e\left(1-\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d+e}\right)}{2\sqrt{-c^2}e}-\frac{bc\log(d+ex)\log\left(-\frac{e\left(\sqrt{-\sqrt{-c^2}}x+1\right)}{\sqrt{-\sqrt{-c^2}}d-e}\right)}{2\sqrt{-c^2}e}$$

```
[In] Int[(a + b*ArcTan[c*x^2])/(d + e*x),x]
```

```
[Out] ((a + b*ArcTan[c*x^2])*Log[d + e*x])/e + (b*c*Log[(e*(1 - (-c^2)^(1/4)*x))/
((-c^2)^(1/4)*d + e)]*Log[d + e*x])/(2*Sqrt[-c^2]*e) + (b*c*Log[-((e*(1 + (
-c^2)^(1/4)*x))/((-c^2)^(1/4)*d - e))]*Log[d + e*x])/(2*Sqrt[-c^2]*e) - (b*
c*Log[(e*(1 - Sqrt[-Sqrt[-c^2]]*x))/(Sqrt[-Sqrt[-c^2]]*d + e)]*Log[d + e*x]
```

```
)/(2*Sqrt[-c^2]*e) - (b*c*Log[-((e*(1 + Sqrt[-Sqrt[-c^2]]*x))/(Sqrt[-Sqrt[-
c^2]]*d - e))]*Log[d + e*x])/(2*Sqrt[-c^2]*e) + (b*c*PolyLog[2, ((-c^2)^(1/
4)*(d + e*x))/((-c^2)^(1/4)*d - e)])/(2*Sqrt[-c^2]*e) - (b*c*PolyLog[2, (Sq
rt[-Sqrt[-c^2]]*(d + e*x))/(Sqrt[-Sqrt[-c^2]]*d - e)])/(2*Sqrt[-c^2]*e) + (
b*c*PolyLog[2, ((-c^2)^(1/4)*(d + e*x))/((-c^2)^(1/4)*d + e)])/(2*Sqrt[-c^2
]*e) - (b*c*PolyLog[2, (Sqrt[-Sqrt[-c^2]]*(d + e*x))/(Sqrt[-Sqrt[-c^2]]*d +
 e)])/(2*Sqrt[-c^2]*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*((a + b*ArcTan[c*x^n])/e), x] - Dist[b*c*(n/e), Int[x^
(n - 1)*(Log[d + e*x]/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, n
}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} - \frac{(2bc)\int \frac{x\log(d+ex)}{1+c^2x^4}\,dx}{e} \\
&= \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} - \frac{(2bc)\int \left(-\frac{c^2x\log(d+ex)}{2\sqrt{-c^2}\left(\sqrt{-c^2}-c^2x^2\right)} - \frac{c^2x\log(d+ex)}{2\sqrt{-c^2}\left(\sqrt{-c^2}+c^2x^2\right)}\right)dx}{e} \\
&= \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} - \frac{\left(bc\sqrt{-c^2}\right)\int \frac{x\log(d+ex)}{\sqrt{-c^2}-c^2x^2}\,dx}{e} - \frac{\left(bc\sqrt{-c^2}\right)\int \frac{x\log(d+ex)}{\sqrt{-c^2}+c^2x^2}\,dx}{e} \\
&= \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} \\
&\quad - \frac{\left(bc\sqrt{-c^2}\right)\int \left(-\frac{\sqrt[4]{-c^2}\log(d+ex)}{2c^2\left(1-\sqrt[4]{-c^2}x\right)} + \frac{\sqrt[4]{-c^2}\log(d+ex)}{2c^2\left(1+\sqrt[4]{-c^2}x\right)}\right)dx}{e} \\
&\quad - \frac{\left(bc\sqrt{-c^2}\right)\int \left(\frac{\sqrt{-\sqrt{-c^2}}\log(d+ex)}{2c^2\left(1-\sqrt{-\sqrt{-c^2}}x\right)} - \frac{\sqrt{-\sqrt{-c^2}}\log(d+ex)}{2c^2\left(1+\sqrt{-\sqrt{-c^2}}x\right)}\right)dx}{e} \\
&= \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} - \frac{(bc)\int \frac{\log(d+ex)}{1-\sqrt[4]{-c^2}x}\,dx}{2\sqrt[4]{-c^2}e} \\
&\quad + \frac{(bc)\int \frac{\log(d+ex)}{1+\sqrt[4]{-c^2}x}\,dx}{2\sqrt[4]{-c^2}e} - \frac{(bc)\int \frac{\log(d+ex)}{1-\sqrt{-\sqrt{-c^2}}x}\,dx}{2\sqrt{-\sqrt{-c^2}}e} + \frac{(bc)\int \frac{\log(d+ex)}{1+\sqrt{-\sqrt{-c^2}}x}\,dx}{2\sqrt{-\sqrt{-c^2}}e}
\end{aligned}$$

$$= \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} + \frac{bc\log\left(\frac{e\left(1-\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$+ \frac{bc\log\left(-\frac{e\left(1+\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(\frac{e\left(1-\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$- \frac{bc\log\left(-\frac{e\left(1+\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$- \frac{(bc)\int \frac{\log\left(\frac{e\left(1-\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d+e}\right)}{d+ex}dx}{2\sqrt{-c^2}} - \frac{(bc)\int \frac{\log\left(\frac{e\left(1+\sqrt[4]{-c^2}x\right)}{-\sqrt[4]{-c^2}d+e}\right)}{d+ex}dx}{2\sqrt{-c^2}}$$

$$+ \frac{(bc)\int \frac{\log\left(\frac{e\left(1-\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d+e}\right)}{d+ex}dx}{2\sqrt{-c^2}} + \frac{(bc)\int \frac{\log\left(\frac{e\left(1+\sqrt{-\sqrt{-c^2}}x\right)}{-\sqrt{-\sqrt{-c^2}}d+e}\right)}{d+ex}dx}{2\sqrt{-c^2}}$$

$$= \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} + \frac{bc\log\left(\frac{e\left(1-\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$+ \frac{bc\log\left(-\frac{e\left(1+\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(\frac{e\left(1-\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$- \frac{bc\log\left(-\frac{e\left(1+\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$- \frac{(bc)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{-c^2}x}{-\sqrt[4]{-c^2}d+e}\right)}{x}dx, x, d+ex\right)}{2\sqrt{-c^2}e}$$

$$- \frac{(bc)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{-c^2}x}{\sqrt[4]{-c^2}d+e}\right)}{x}dx, x, d+ex\right)}{2\sqrt{-c^2}e}$$

$$+ \frac{(bc)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-\sqrt{-c^2}}x}{-\sqrt{-\sqrt{-c^2}}d+e}\right)}{x}dx, x, d+ex\right)}{2\sqrt{-c^2}e}$$

$$+ \frac{(bc)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-\sqrt{-c^2}}x}{\sqrt{-\sqrt{-c^2}}d+e}\right)}{x}dx, x, d+ex\right)}{2\sqrt{-c^2}e}$$

$$= \frac{(a+b\arctan(cx^2))\log(d+ex)}{e} + \frac{bc\log\left(\frac{e\left(1-\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$+ \frac{bc\log\left(-\frac{e\left(1+\sqrt[4]{-c^2}x\right)}{\sqrt[4]{-c^2}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(\frac{e\left(1-\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$- \frac{bc\log\left(-\frac{e\left(1+\sqrt{-\sqrt{-c^2}}x\right)}{\sqrt{-\sqrt{-c^2}}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$+ \frac{bc\,\mathrm{PolyLog}\left(2, \frac{\sqrt[4]{-c^2}(d+ex)}{\sqrt[4]{-c^2}d-e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\mathrm{PolyLog}\left(2, \frac{\sqrt{-\sqrt{-c^2}}(d+ex)}{\sqrt{-\sqrt{-c^2}}d-e}\right)}{2\sqrt{-c^2}e}$$

$$+ \frac{bc\,\mathrm{PolyLog}\left(2, \frac{\sqrt[4]{-c^2}(d+ex)}{\sqrt[4]{-c^2}d+e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\mathrm{PolyLog}\left(2, \frac{\sqrt{-\sqrt{-c^2}}(d+ex)}{\sqrt{-\sqrt{-c^2}}d+e}\right)}{2\sqrt{-c^2}e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.06 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.65

$$\int \frac{a+b\arctan(cx^2)}{d+ex}\,dx = \frac{a\log(d+ex)}{e}$$
$$+ \frac{b\left(2\arctan(cx^2)\log(d+ex) + i\left(\log(d+ex)\log\left(1-\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt[4]{-1}e}\right) + \log(d+ex)\log\left(1-\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt[4]{-1}e}\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(2*ArcTan[c*x^2]*Log[d + e*x] + I*(Log[d + e*x]*Log
[1 - (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - (-1)^(1/4)*e)] + Log[d + e*x]*Log[1 -
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + (-1)^(1/4)*e)] - Log[d + e*x]*Log[1 - (Sq
rt[c]*(d + e*x))/(Sqrt[c]*d - (-1)^(3/4)*e)] - Log[d + e*x]*Log[1 - (Sqrt[c
]*(d + e*x))/(Sqrt[c]*d + (-1)^(3/4)*e)] + PolyLog[2, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - (-1)^(1/4)*e)] + PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + (-
1)^(1/4)*e)] - PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - (-1)^(3/4)*e)] -
 PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + (-1)^(3/4)*e)])))/(2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.28

method	result
default	$\frac{a\ln(ex+d)}{e}+\frac{b\ln(ex+d)\arctan(c\,x^2)}{e}-\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2\_Z^4-4c^2d\_Z^3+6c^2d^2\_Z^2-4\_Z c^2d^3+c^2d^4+e^4\right)}\frac{\ln(ex+d)\ln\left(\frac{-ex+\_R}{\_R}\right.}{\_R1}\right.}{2c}$
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\ln(ex+d)\arctan(c\,x^2)}{e}-\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2\_Z^4-4c^2d\_Z^3+6c^2d^2\_Z^2-4\_Z c^2d^3+c^2d^4+e^4\right)}\frac{\ln(ex+d)\ln\left(\frac{-ex+\_R}{\_R}\right.}{\_R1}\right.}{2c}$
risch	$\frac{ib\ln(ex+d)\ln(-ic\,x^2+1)}{2e}-\frac{ib\ln(ex+d)\ln\left(\frac{e\sqrt{-ic}-c(ex+d)+cd}{e\sqrt{-ic}+cd}\right)}{2e}-\frac{ib\ln(ex+d)\ln\left(\frac{e\sqrt{-ic}+c(ex+d)-cd}{e\sqrt{-ic}-cd}\right)}{2e}-\frac{ib\,\mathrm{dilog}\left(\frac{e\sqrt{-ic}-c(ex+d}{e\sqrt{-ic}+cd}\right.}{2e}$

```
[In] int((a+b*arctan(c*x^2))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(e*x+d)/e+b*ln(e*x+d)/e*arctan(c*x^2)-1/2*b/c*e*sum(1/(_R1^2-2*_R1*d+d^
2)*(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1)),_R1=RootOf(_Z^4
*c^2-4*_Z^3*c^2*d+6*_Z^2*c^2*d^2-4*_Z*c^2*d^3+c^2*d^4+e^4))
```

Fricas [F]

$$\int \frac{a+b\arctan(cx^2)}{d+ex}\,dx=\int \frac{b\arctan(cx^2)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^2) + a)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx^2)}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*atan(c*x**2))/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a + b \arctan(cx^2)}{d + ex} \, dx = \int \frac{b \arctan(cx^2) + a}{ex + d} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d),x, algorithm="maxima")
[Out] 2*b*integrate(1/2*arctan(c*x^2)/(e*x + d), x) + a*log(e*x + d)/e
```

Giac [F]

$$\int \frac{a + b \arctan(cx^2)}{d + ex} \, dx = \int \frac{b \arctan(cx^2) + a}{ex + d} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx^2)}{d + ex} \, dx = \int \frac{a + b \operatorname{atan}(c x^2)}{d + e x} \, dx$$

```
[In] int((a + b*atan(c*x^2))/(d + e*x),x)
[Out] int((a + b*atan(c*x^2))/(d + e*x), x)
```

3.24 $\int \frac{a+b\arctan(cx^2)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 328

$$\begin{aligned}\int \frac{a+b\arctan(cx^2)}{(d+ex)^2}\,dx = {} & \frac{bc^2d^3\arctan(cx^2)}{e(c^2d^4+e^4)} - \frac{a+b\arctan(cx^2)}{e(d+ex)} \\ & + \frac{b\sqrt{c}(cd^2-e^2)\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\,(c^2d^4+e^4)} \\ & - \frac{b\sqrt{c}(cd^2-e^2)\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\,(c^2d^4+e^4)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} \\ & - \frac{b\sqrt{c}(cd^2+e^2)\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\,(c^2d^4+e^4)} \\ & + \frac{b\sqrt{c}(cd^2+e^2)\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\,(c^2d^4+e^4)} + \frac{bcde\log(1+c^2x^4)}{2(c^2d^4+e^4)}\end{aligned}$$

```
[Out] b*c^2*d^3*arctan(c*x^2)/e/(c^2*d^4+e^4)+(-a-b*arctan(c*x^2))/e/(e*x+d)-2*b*
c*d*e*ln(e*x+d)/(c^2*d^4+e^4)+1/2*b*c*d*e*ln(c^2*x^4+1)/(c^2*d^4+e^4)-1/2*b
*(c*d^2-e^2)*arctan(-1+x*2^(1/2)*c^(1/2))*c^(1/2)/(c^2*d^4+e^4)*2^(1/2)-1/2
*b*(c*d^2-e^2)*arctan(1+x*2^(1/2)*c^(1/2))*c^(1/2)/(c^2*d^4+e^4)*2^(1/2)-1/
4*b*(c*d^2+e^2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*c^(1/2)/(c^2*d^4+e^4)*2^(1/2)
+1/4*b*(c*d^2+e^2)*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*c^(1/2)/(c^2*d^4+e^4)*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {4980, 6857, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$\int \frac{a+b\arctan\left(cx^2\right)}{(d+ex)^2}\,dx = -\frac{a+b\arctan\left(cx^2\right)}{e(d+ex)}+\frac{bc^2d^3\arctan\left(cx^2\right)}{e\left(c^2d^4+e^4\right)}+\frac{b\sqrt{c}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)\left(cd^2-e^2\right)}{\sqrt{2}\left(c^2d^4+e^4\right)}-\frac{b\sqrt{c}\arctan\left(\sqrt{2}\sqrt{c}x+1\right)\left(cd^2-e^2\right)}{\sqrt{2}\left(c^2d^4+e^4\right)}+\frac{bcde\log\left(c^2x^4+1\right)}{2\left(c^2d^4+e^4\right)}-\frac{2bcde\log(d+ex)}{c^2d^4+e^4}-\frac{b\sqrt{c}(cd^2+e^2)\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\left(c^2d^4+e^4\right)}+\frac{b\sqrt{c}(cd^2+e^2)\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\left(c^2d^4+e^4\right)}$$

```
[In] Int[(a + b*ArcTan[c*x^2])/(d + e*x)^2,x]
```

```
[Out] (b*c^2*d^3*ArcTan[c*x^2])/(e*(c^2*d^4 + e^4)) - (a + b*ArcTan[c*x^2])/(e*(d
 + e*x)) + (b*Sqrt[c]*(c*d^2 - e^2)*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]
*(c^2*d^4 + e^4)) - (b*Sqrt[c]*(c*d^2 - e^2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])
/(Sqrt[2]*(c^2*d^4 + e^4)) - (2*b*c*d*e*Log[d + e*x])/(c^2*d^4 + e^4) - (b*
Sqrt[c]*(c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*(c^2*d
^4 + e^4)) + (b*Sqrt[c]*(c*d^2 + e^2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(
2*Sqrt[2]*(c^2*d^4 + e^4)) + (b*c*d*e*Log[1 + c^2*x^4])/(2*(c^2*d^4 + e^4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_) + (e_.)*(x_))^(m_.), x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(e*(m + 1))), x] - D
ist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 + c^2*x^(2*n))
), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{a+b\arctan\left(cx^2\right)}{e(d+ex)}+\frac{(2bc)\int \frac{x}{(d+ex)\left(1+c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a+b\arctan\left(cx^2\right)}{e(d+ex)}+\frac{(2bc)\int \left(-\frac{de^3}{\left(c^2d^4+e^4\right)(d+ex)}+\frac{e^3+c^2d^3x-c^2d^2ex^2+c^2de^2x^3}{\left(c^2d^4+e^4\right)\left(1+c^2x^4\right)}\right)dx}{e}$$

$$= -\frac{a+b\arctan\left(cx^2\right)}{e(d+ex)}-\frac{2bcde\log(d+ex)}{c^2d^4+e^4}+\frac{(2bc)\int \frac{e^3+c^2d^3x-c^2d^2ex^2+c^2de^2x^3}{1+c^2x^4}\,dx}{e\left(c^2d^4+e^4\right)}$$

$$= -\frac{a+b\arctan\left(cx^2\right)}{e(d+ex)}-\frac{2bcde\log(d+ex)}{c^2d^4+e^4}+\frac{(2bc)\int \left(\frac{e^3-c^2d^2ex^2}{1+c^2x^4}+\frac{x\left(c^2d^3+c^2de^2x^2\right)}{1+c^2x^4}\right)dx}{e\left(c^2d^4+e^4\right)}$$

$$= -\frac{a+b\arctan\left(cx^2\right)}{e(d+ex)}-\frac{2bcde\log(d+ex)}{c^2d^4+e^4}+\frac{(2bc)\int \frac{e^3-c^2d^2ex^2}{1+c^2x^4}\,dx}{e\left(c^2d^4+e^4\right)}+\frac{(2bc)\int \frac{x\left(c^2d^3+c^2de^2x^2\right)}{1+c^2x^4}\,dx}{e\left(c^2d^4+e^4\right)}$$

$$= -\frac{a+b\arctan\left(cx^2\right)}{e(d+ex)}-\frac{2bcde\log(d+ex)}{c^2d^4+e^4}+\frac{(bc)\text{Subst}\left(\int \frac{c^2d^3+c^2de^2x}{1+c^2x^2}\,dx,x,x^2\right)}{e\left(c^2d^4+e^4\right)} - \frac{\left(b\left(cd^2-e^2\right)\right)\int \frac{c+c^2x^2}{1+c^2x^4}\,dx}{c^2d^4+e^4}+\frac{\left(b\left(cd^2+e^2\right)\right)\int \frac{c-c^2x^2}{1+c^2x^4}\,dx}{c^2d^4+e^4}$$

$$= -\frac{a+b\arctan(cx^2)}{e(d+ex)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} + \frac{(bc^3d^3)\,\mathrm{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{e(c^2d^4+e^4)} + \frac{(bc^3de)\,\mathrm{Subst}\left(\int\frac{x}{1+c^2x^2}\,dx,x,x^2\right)}{c^2d^4+e^4} - \frac{(b(cd^2-e^2))\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{2(c^2d^4+e^4)} - \frac{(b(cd^2-e^2))\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{2(c^2d^4+e^4)} - \frac{\left(b\sqrt{c}(cd^2+e^2)\right)\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}(c^2d^4+e^4)} - \frac{\left(b\sqrt{c}(cd^2+e^2)\right)\int\frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}(c^2d^4+e^4)}$$

$$= \frac{bc^2d^3\arctan(cx^2)}{e(c^2d^4+e^4)} - \frac{a+b\arctan(cx^2)}{e(d+ex)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} - \frac{b\sqrt{c}(cd^2+e^2)\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}(c^2d^4+e^4)} + \frac{b\sqrt{c}(cd^2+e^2)\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}(c^2d^4+e^4)} + \frac{bcde\log(1+c^2x^4)}{2(c^2d^4+e^4)} - \frac{\left(b\sqrt{c}(cd^2-e^2)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}(c^2d^4+e^4)} + \frac{\left(b\sqrt{c}(cd^2-e^2)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}(c^2d^4+e^4)}$$

$$= \frac{bc^2d^3\arctan(cx^2)}{e(c^2d^4+e^4)} - \frac{a+b\arctan(cx^2)}{e(d+ex)} + \frac{b\sqrt{c}(cd^2-e^2)\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}(c^2d^4+e^4)} - \frac{b\sqrt{c}(cd^2-e^2)\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}(c^2d^4+e^4)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} - \frac{b\sqrt{c}(cd^2+e^2)\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}(c^2d^4+e^4)} + \frac{b\sqrt{c}(cd^2+e^2)\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}(c^2d^4+e^4)} + \frac{bcde\log(1+c^2x^4)}{2(c^2d^4+e^4)}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.98

$$\int\frac{a+b\arctan(cx^2)}{(d+ex)^2}\,dx = -\frac{4a(c^2d^4+e^4)+4b(c^2d^4+e^4)\arctan(cx^2)+2b\sqrt{c}\left(2c^{3/2}d^3-\sqrt{2}cd^2e+\sqrt{2}e^3\right)(d+ex)\arctan\left(1-\sqrt{2}\sqrt{\right.}}{}$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/(d + e*x)^2,x]
```

```
[Out] -1/4*(4*a*(c^2*d^4 + e^4) + 4*b*(c^2*d^4 + e^4)*ArcTan[c*x^2] + 2*b*Sqrt[c]
*(2*c^(3/2)*d^3 - Sqrt[2]*c*d^2*e + Sqrt[2]*e^3)*(d + e*x)*ArcTan[1 - Sqrt[
```

```
2]*Sqrt[c]*x] + 2*b*Sqrt[c]*(2*c^(3/2)*d^3 + Sqrt[2]*c*d^2*e - Sqrt[2]*e^3)
*(d + e*x)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x] + 8*b*c*d*e^2*(d + e*x)*Log[d + e*
x] + Sqrt[2]*b*Sqrt[c]*e*(c*d^2 + e^2)*(d + e*x)*Log[1 - Sqrt[2]*Sqrt[c]*x
+ c*x^2] - Sqrt[2]*b*Sqrt[c]*e*(c*d^2 + e^2)*(d + e*x)*Log[1 + Sqrt[2]*Sqrt
[c]*x + c*x^2] - 2*b*c*d*e^2*(d + e*x)*Log[1 + c^2*x^4])/(e*(c^2*d^4 + e^4)
*(d + e*x))
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a}{(ex+d)e}+b\left(-\frac{\arctan(c\,x^2)}{(ex+d)e}+\frac{2c\left(-\frac{d\,e^2\ln(ex+d)}{c^2d^4+e^4}+\frac{\frac{e^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\right.}{8}}{}\right.}{}\right.$
parts	$-\frac{a}{(ex+d)e}+b\left(-\frac{\arctan(c\,x^2)}{(ex+d)e}+\frac{2c\left(-\frac{d\,e^2\ln(ex+d)}{c^2d^4+e^4}+\frac{\frac{e^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\right.}{8}}{}\right.}{}\right.$

```
[In] int((a+b*arctan(c*x^2))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(e*x+d)/e+b*(-1/(e*x+d)/e*arctan(c*x^2)+2*c/e*(-d*e^2/(c^2*d^4+e^4)*ln(e
*x+d)+1/(c^2*d^4+e^4)*(1/8*e^3*(1/c^2)^(1/4)*2^(1/2)*(ln((x^2+(1/c^2)^(1/4)
*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*ar
ctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1))+1/2*c^
2*d^3/(c^2)^(1/2)*arctan(x^2*(c^2)^(1/2))-1/8*d^2*e/(1/c^2)^(1/4)*2^(1/2)*(
ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)
+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^
2)^(1/4)*x-1))+1/4*d*e^2*ln(c^2*x^4+1))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 74.55 (sec) , antiderivative size = 2478078, normalized size of antiderivative = 7555.12

$$\int \frac{a+b\arctan\left(cx^2\right)}{(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan\left(cx^2\right)}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(c*x**2))/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan\left(cx^2\right)}{(d+ex)^2}\,dx =$$

$$-\frac{1}{4}\left(\left(\left(\frac{8\,de\log\left(ex+d\right)}{c^2d^4+e^4} - \frac{\frac{\sqrt{2}\left(cd^2e+\sqrt{2}\sqrt{c}de^2+e^3\right)\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}} - \frac{\sqrt{2}\left(cd^2e-\sqrt{2}\sqrt{c}de^2+e^3\right)\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}} - \frac{2}{}}{c}\right.\right.\right.$$

$$-\frac{a}{e^2x+de}$$

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((8*d*e*log(e*x + d)/(c^2*d^4 + e^4) - (sqrt(2)*(c*d^2*e + sqrt(2)*sqr
t(c)*d*e^2 + e^3)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*(c*d
^2*e - sqrt(2)*sqrt(c)*d*e^2 + e^3)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/sqrt
(c) - 2*(2*c^2*d^3 + sqrt(2)*c^(3/2)*d^2*e - sqrt(2)*sqrt(c)*e^3)*arctan(1/
2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c + 2*(2*c^2*d^3 - sqrt(2)*c^(
3/2)*d^2*e + sqrt(2)*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(
c))/sqrt(c))/c)/(c^2*d^4*e + e^5))*c + 4*arctan(c*x^2)/(e^2*x + d*e))*b - a
/(e^2*x + d*e)
```

Giac [F]

$$\int \frac{a+b\arctan\left(cx^2\right)}{(d+ex)^2}\,dx=\int \frac{b\arctan\left(cx^2\right)+a}{\left(ex+d\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 883, normalized size of antiderivative = 2.69

$$\int \frac{a+b\arctan\left(cx^2\right)}{(d+ex)^2}\,dx$$
$$=\left(\sum_{k=1}^{4}\ln\left(\frac{\mathrm{root}(16\,c^2\,d^4\,e^4\,z^4+16\,e^8\,z^4-32\,b\,c\,d\,e^5\,z^3+8\,b^2\,c^2\,d^2\,e^2\,z^2+b^4\,c^2,z,k)^4\,c^8\,e^9\,x\,320-\mathrm{root}(16\,c}{}\right.\right.$$
$$\left.\left.+16\,e^8\,z^4-32\,b\,c\,d\,e^5\,z^3+8\,b^2\,c^2\,d^2\,e^2\,z^2+b^4\,c^2,z,k)\right)\right)$$
$$-\frac{a}{x\,e^2+d\,e}-\frac{b\,\mathrm{atan}(c\,x^2)}{x\,e^2+d\,e}-\frac{2\,b\,c\,d\,e\,\ln\left(d+e\,x\right)}{c^2\,d^4+e^4}$$

```
[In] int((a + b*atan(c*x^2))/(d + e*x)^2,x)
```

```
[Out] symsum(log((320*root(16*c^2*d^4*e^4*z^4 + 16*e^8*z^4 - 32*b*c*d*e^5*z^3 + 8
*b^2*c^2*d^2*e^2*z^2 + b^4*c^2, z, k)^4*c^8*e^9*x - 128*root(16*c^2*d^4*e^4
*z^4 + 16*e^8*z^4 - 32*b*c*d*e^5*z^3 + 8*b^2*c^2*d^2*e^2*z^2 + b^4*c^2, z,
k)^4*c^10*d^5*e^4 + 16*b^4*c^10*e*x - 8*root(16*c^2*d^4*e^4*z^4 + 16*e^8*z^
4 - 32*b*c*d*e^5*z^3 + 8*b^2*c^2*d^2*e^2*z^2 + b^4*c^2, z, k)*b^3*c^9*e^3 +
 384*root(16*c^2*d^4*e^4*z^4 + 16*e^8*z^4 - 32*b*c*d*e^5*z^3 + 8*b^2*c^2*d^
2*e^2*z^2 + b^4*c^2, z, k)^4*c^8*d*e^8 + 8*root(16*c^2*d^4*e^4*z^4 + 16*e^8
*z^4 - 32*b*c*d*e^5*z^3 + 8*b^2*c^2*d^2*e^2*z^2 + b^4*c^2, z, k)*b^3*c^11*d
```

```
^3*x - 320*root(16*c^2*d^4*e^4*z^4 + 16*e^8*z^4 - 32*b*c*d*e^5*z^3 + 8*b^2*
c^2*d^2*e^2*z^2 + b^4*c^2, z, k)^3*b*c^9*d^2*e^5 - 192*root(16*c^2*d^4*e^4*
z^4 + 16*e^8*z^4 - 32*b*c*d*e^5*z^3 + 8*b^2*c^2*d^2*e^2*z^2 + b^4*c^2, z, k
)^4*c^10*d^4*e^5*x + 32*root(16*c^2*d^4*e^4*z^4 + 16*e^8*z^4 - 32*b*c*d*e^5
*z^3 + 8*b^2*c^2*d^2*e^2*z^2 + b^4*c^2, z, k)^3*b*c^11*d^5*e^2*x + 64*root(
16*c^2*d^4*e^4*z^4 + 16*e^8*z^4 - 32*b*c*d*e^5*z^3 + 8*b^2*c^2*d^2*e^2*z^2
+ b^4*c^2, z, k)^2*b^2*c^10*d^2*e^3*x - 416*root(16*c^2*d^4*e^4*z^4 + 16*e^
8*z^4 - 32*b*c*d*e^5*z^3 + 8*b^2*c^2*d^2*e^2*z^2 + b^4*c^2, z, k)^3*b*c^9*d
*e^6*x)/e^2)*root(16*c^2*d^4*e^4*z^4 + 16*e^8*z^4 - 32*b*c*d*e^5*z^3 + 8*b^
2*c^2*d^2*e^2*z^2 + b^4*c^2, z, k), k, 1, 4) - a/(d*e + e^2*x) - (b*atan(c*
x^2))/(d*e + e^2*x) - (2*b*c*d*e*log(d + e*x))/(e^4 + c^2*d^4)
```

3.25 $\int (d+ex) \left(a+b \arctan \left(cx^2\right)\right)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 1325

$$\int (d+ex) \left(a+b \arctan \left(cx^2\right)\right)^2 \, dx = a^2dx - \frac{2(-1)^{3/4}abd \arctan \left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}$$

$$+ \frac{(-1)^{3/4}b^2d \arctan \left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{ie\left(a+b \arctan \left(cx^2\right)\right)^2}{2c}$$

$$+\frac{1}{2}ex^2\left(a+b \arctan \left(cx^2\right)\right)^2+\frac{2(-1)^{3/4}abd\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2d\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2\sqrt[4]{-}}{}$$

```
[Out] -(-1)^(1/4)*b^2*d*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(1/2)+(-1)^(
1/4)*b^2*d*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(1/2)+(-1)^(1/4)*b
^2*d*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)
^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*d*arctanh((-1)^(3/4)*x*c^(1/2))*l
n(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1
/4)*b^2*d*arctanh((-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(
1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*d*arctan((-1)^(3/4)*x*c^(1/
2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+1/2
*I*e*(a+b*arctan(c*x^2))^2/c+b*e*(a+b*arctan(c*x^2))*ln(2/(1+I*c*x^2))/c+(-
1)^(3/4)*b^2*d*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(1/2)-(-1)^(1/4)*b^2*d*arct
anh((-1)^(3/4)*x*c^(1/2))^2/c^(1/2)+(-1)^(3/4)*b^2*d*polylog(2,1-2/(1-(-1)^
(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(3/4)*b^2*d*polylog(2,1-2/(1+(-1)^(1/4)*x*c^
(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*d*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))/c^
(1/2)+(-1)^(1/4)*b^2*d*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+1/2*
e*x^2*(a+b*arctan(c*x^2))^2+1/2*b^2*d*x*ln(1-I*c*x^2)*ln(1+I*c*x^2)-1/2*(-1
)^(3/4)*b^2*d*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^
(1/2)))/c^(1/2)-1/2*(-1)^(1/4)*b^2*d*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1
/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-1/2*(-1)^(1/4)*b^2*d*polylog(2,1-(1+
```

```
I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-1/2*(-1)^(3/4
)*b^2*d*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)
))/c^(1/2)-1/4*b^2*d*x*ln(1-I*c*x^2)^2-1/4*b^2*d*x*ln(1+I*c*x^2)^2+a^2*d*x+
1/2*I*b^2*e*polylog(2,1-2/(1+I*c*x^2))/c+I*a*b*d*x*ln(1-I*c*x^2)+(-1)^(1/4)
*b^2*d*arctan((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(1/2)-(-1)^(1/4)*b^2*d*
arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(1/2)-2*(-1)^(3/4)*a*b*d*arct
an((-1)^(3/4)*x*c^(1/2))/c^(1/2)+2*(-1)^(3/4)*a*b*d*arctanh((-1)^(3/4)*x*c^
(1/2))/c^(1/2)+2*(-1)^(1/4)*b^2*d*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)
^(1/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(1/4)*b^2*d*arctan((-1)^(3/4)*x*c^(1/2))*
ln(2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+2*(-1)^(1/4)*b^2*d*arctanh((-1)^(3/4
)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(1/4)*b^2*d*arct
anh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-I*a*b*d*x*
ln(1+I*c*x^2)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 1325, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.444$, Rules used = {4982, 4932, 2498, 327, 209, 2500, 2526, 2520, 12, 5040, 4964, 2449, 2352, 212, 2636, 211, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055, 4948, 4930}

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$= dxa^2 - \frac{2(-1)^{3/4}bd\arctan\left((-1)^{3/4}\sqrt{c}x\right)a}{\sqrt{c}} + \frac{2(-1)^{3/4}bd\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)a}{\sqrt{c}}$$
$$+ibdx\log\left(1-icx^2\right)a-ibdx\log\left(icx^2+1\right)a+\frac{(-1)^{3/4}b^2d\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2$$

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] a^2*d*x - (2*(-1)^(3/4)*a*b*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)
^(3/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + ((I/2)*e*(a + b*ArcT
an[c*x^2])^2)/c + (e*x^2*(a + b*ArcTan[c*x^2])^2)/2 + (2*(-1)^(3/4)*a*b*d*A
rcTanh[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4
)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*
Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*d*ArcTan[(-1
)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)
*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/
(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/
4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^
2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c
] + ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3
/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*d
```

```
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/Sqrt[c] + I*a*b*d*x*Log[1 - I*c*x^2] + ((-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/
4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/Sqrt[c] - ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/Sqrt[c] - (b^2*d*x*Log[1 - I*c*x^2]^2)/4 +
 (b*e*(a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c - I*a*b*d*x*Log[1 + I*c
*x^2] - ((-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sq
rt[c] + ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/S
qrt[c] + (b^2*d*x*Log[1 - I*c*x^2]*Log[1 + I*c*x^2])/2 - (b^2*d*x*Log[1 + I
*c*x^2]^2)/4 + ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x
)])/Sqrt[c] + ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/Sqrt[c] - ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c
]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sqrt[c]) + ((-1)^(1/4)*b^2*d*PolyLog[
2, 1 - 2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*d*PolyLog[2
, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(1/4)*b^2*d*PolyLog[2,
 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqr
t[c]) - ((-1)^(1/4)*b^2*d*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x
))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqrt[c]) - ((-1)^(3/4)*b^2*d*PolyLog[2,
1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sq
rt[c]) + ((I/2)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2636

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[x*Log[w]*(D[v, x]/v), x], x] - Int[SimplifyIntegrand[x*Log[v]*(D[
w, x]/w), x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4932

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandIn
tegrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4982

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x^n])^p, (d + e*x)^m, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 1] && IGtQ[m, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d\left(a+b\arctan\left(cx^2\right)\right)^2+ex\left(a+b\arctan\left(cx^2\right)\right)^2\right)\,dx \\
&= d\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx+e\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx \\
&= d\int \Bigg(a^2+iab\log\left(1-icx^2\right)-\frac{1}{4}b^2\log^2\left(1-icx^2\right)-iab\log\left(1+icx^2\right) \\
&\qquad +\frac{1}{2}b^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)-\frac{1}{4}b^2\log^2\left(1+icx^2\right)\Bigg)\,dx \\
&\quad +\frac{1}{2}e\text{Subst}\left(\int (a+b\arctan(cx))^2\,dx,x,x^2\right) \\
&= a^2dx+\frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2+(iabd)\int \log\left(1-icx^2\right)\,dx \\
&\quad -(iabd)\int \log\left(1+icx^2\right)\,dx-\frac{1}{4}\left(b^2d\right)\int \log^2\left(1-icx^2\right)\,dx \\
&\quad -\frac{1}{4}\left(b^2d\right)\int \log^2\left(1+icx^2\right)\,dx+\frac{1}{2}\left(b^2d\right)\int \log\left(1-icx^2\right)\log\left(1+icx^2\right)\,dx \\
&\quad -(bce)\text{Subst}\left(\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^2\right) \\
&= a^2dx+\frac{ie\left(a+b\arctan\left(cx^2\right)\right)^2}{2c}+\frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2 \\
&\quad +iabdx\log\left(1-icx^2\right)-\frac{1}{4}b^2dx\log^2\left(1-icx^2\right)-iabdx\log\left(1+icx^2\right) \\
&\quad +\frac{1}{2}b^2dx\log\left(1-icx^2\right)\log\left(1+icx^2\right)-\frac{1}{4}b^2dx\log^2\left(1+icx^2\right) \\
&\quad -\frac{1}{2}\left(b^2d\right)\int \frac{2cx^2\log\left(1-icx^2\right)}{-i+cx^2}\,dx-\frac{1}{2}\left(b^2d\right)\int \frac{2cx^2\log\left(1+icx^2\right)}{i+cx^2}\,dx \\
&\quad -(2abcd)\int \frac{x^2}{1-icx^2}\,dx-(2abcd)\int \frac{x^2}{1+icx^2}\,dx-\left(ib^2cd\right)\int \frac{x^2\log\left(1-icx^2\right)}{1-icx^2}\,dx \\
&\quad +\left(ib^2cd\right)\int \frac{x^2\log\left(1+icx^2\right)}{1+icx^2}\,dx+(be)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{i-cx}\,dx,x,x^2\right)
\end{aligned}$$

$$= a^2dx + \frac{ie(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2 + iabdx\log\left(1-icx^2\right) - \frac{1}{4}b^2dx\log^2\left(1-icx^2\right) + \frac{be(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c} - iabdx\log\left(1+icx^2\right) + \frac{1}{2}b^2dx\log\left(1-icx^2\right)\log\left(1+icx^2\right) - \frac{1}{4}b^2dx\log^2\left(1+icx^2\right) + (2iabd)\int\frac{1}{1-icx^2}\,dx - (2iabd)\int\frac{1}{1+icx^2}\,dx - \left(ib^2cd\right)\int\left(\frac{i\log(1-icx^2)}{c} - \frac{i\log(1-icx^2)}{c(1-icx^2)}\right)dx + \left(ib^2cd\right)\int\left(-\frac{i\log(1+icx^2)}{c} + \frac{i\log(1+icx^2)}{c(1+icx^2)}\right)dx - \left(b^2cd\right)\int\frac{x^2\log(1-icx^2)}{-i+cx^2}\,dx - \left(b^2cd\right)\int\frac{x^2\log(1+icx^2)}{i+cx^2}\,dx - \left(b^2e\right)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)$$

$$= a^2dx - \frac{2(-1)^{3/4}abd\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{ie(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2 + \frac{2(-1)^{3/4}abd\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + iabdx\log\left(1-icx^2\right) - \frac{1}{4}b^2dx\log^2\left(1-icx^2\right) + \frac{be(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c} - iabdx\log\left(1+icx\right.$$

$$= a^2dx - \frac{2(-1)^{3/4}abd\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{ie(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2 + \frac{2(-1)^{3/4}abd\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + iabdx\log\left(1-icx^2\right) + b^2dx\log\left(1-icx^2\right) + \frac{\sqrt[4]{-1}b^2d\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}} - \frac{1}{4}b^2dx\log$$

$$= a^2dx - 4b^2dx - \frac{2(-1)^{3/4}abd\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{ie(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2 + \frac{2(-1)^{3/4}abd\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + iabdx\log\left(1-icx^2\right) + \frac{\sqrt[4]{-1}b^2d\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}} - \frac{\sqrt[4]{-1}b^2d\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right.}{\sqrt{c}}$$

$$= a^2dx - \frac{2(-1)^{3/4}abd\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} - \frac{2\sqrt[4]{-1}b^2d\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2d\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{ie(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2 + \frac{2(-1)^{3/4}abd\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} - \frac{2\sqrt[4]{-1}b^2d\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}$$

$$= a^2dx - \frac{2(-1)^{3/4}abd\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2d\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{ie(a+b\arctan(cx^2))^2}{2c}$$
$$+\frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{2(-1)^{3/4}abd\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2d\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}$$

$$= a^2dx - \frac{2(-1)^{3/4}abd\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2d\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{ie(a+b\arctan(cx^2))^2}{2c}$$
$$+\frac{1}{2}ex^2\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{2(-1)^{3/4}abd\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2d\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}$$

$= \text{Too large to display}$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4824 vs. 2(1325) = 2650.

Time = 36.63 (sec) , antiderivative size = 4824, normalized size of antiderivative = 3.64

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)^2 dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*b*d*Sqrt[c*x^2]*(2*Sqrt[c*x^2]*ArcTan[c*x^2] -
 Sqrt[2]*(ArcTan[(-1 + c*x^2)/(Sqrt[2]*Sqrt[c*x^2])] - ArcTanh[(Sqrt[2]*Sqr
t[c*x^2])/(1 + c*x^2)])))/(c*x) + (a*b*e*(c*x^2*ArcTan[c*x^2] + Log[1/Sqrt[
1 + c^2*x^4]]))/c + (b^2*e*((-I)*ArcTan[c*x^2]^2 + c*x^2*ArcTan[c*x^2]^2 +
2*ArcTan[c*x^2]*Log[1 + E^((2*I)*ArcTan[c*x^2])] - I*PolyLog[2, -E^((2*I)*A
rcTan[c*x^2])]))/(2*c) + (b^2*d*Sqrt[c*x^2]*(2*Sqrt[c*x^2]*ArcTan[c*x^2]^2
- 4*((ArcTan[c*x^2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 2*ArcTan[1 + Sqrt
[2]*Sqrt[c*x^2]] + Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] - Log[1 + c*x^2 + S
qrt[2]*Sqrt[c*x^2]]))/(2*Sqrt[2]) - (-((ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + A
rcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]]) + (Ar
cTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*
x^2 + Sqrt[2]*Sqrt[c*x^2]] - (Sqrt[c*x^2]*(1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2)
^(3/2)*(2*(-5*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 4*ArcTan[1 -
Sqrt[2]*Sqrt[c*x^2]]^2 + ((1 + 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x
^2]]^2)/E^(I*ArcTan[2 + I]) + ((1 - 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqr
```

```
t[c*x^2]]^2)/E^ArcTanh[1 + 2*I] - (5*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Arc
Tanh[1 + 2*I] + (5*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])*Lo
g[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 5*((-
I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTanh[1 + 2*I])*Log[1 - E^((2*I)*Arc
Tan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5*I)*ArcTan[2 + I]*L
og[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]] - 5*ArcTanh[1 + 2
*I]*Log[Sin[ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]]) + 5*Pol
yLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] - 5*P
olyLog[2, E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*
(3 + 2*Cos[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 - Sqrt[2]*
Sqrt[c*x^2]]]))/(20*Sqrt[2]*(-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 +
 Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2] - (1 - Sqrt[
2]*Sqrt[c*x^2])/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2])) + ((1/40 + I/40)*c*
x^2*(1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2)*((5 + 5*I)*Pi*ArcTan[1 - Sqrt[2]*Sqrt
[c*x^2]] + 10*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + (4 - 4*I)*Arc
Tan[1 - Sqrt[2]*Sqrt[c*x^2]]^2 - ((2 + 4*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*
Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) + ((4 + 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqr
t[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I] + 10*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]
]*ArcTanh[1 + 2*I] + (5 - 5*I)*Pi*Log[1 + E^((-2*I)*ArcTan[1 - Sqrt[2]*Sqrt
[c*x^2]])] + (10*I)*ArcTan[2 + I]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan
[1 - Sqrt[2]*Sqrt[c*x^2]]))] - (10*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1
 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 10*ArcTa
n[1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]
 - 2*ArcTanh[1 + 2*I])] + (10*I)*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1
 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] - (5 - 5*I)*Pi*Log[1/Sqrt[1
+ (1 - Sqrt[2]*Sqrt[c*x^2])^2]] - (10*I)*ArcTan[2 + I]*Log[-Sin[ArcTan[2 +
I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]] - (10*I)*ArcTanh[1 + 2*I]*Log[Sin[Ar
cTan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]] - 5*PolyLog[2, E^((2*I
)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] - (5*I)*PolyLog[2, E
^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[
2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]
]]))/((-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*
(1/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2] - (1 - Sqrt[2]*Sqrt[c*x^2])/Sqrt[1
 + (1 - Sqrt[2]*Sqrt[c*x^2])^2])^2) + ((1/80 + I/80)*(2 + 2*c*x^2 - 2*Sqrt[
2]*Sqrt[c*x^2])^2*((-5 - 5*I)*Pi*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - (10*I)*A
rcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + (8 - 8*I)*ArcTan[1 - Sqrt[2]
*Sqrt[c*x^2]]^2 - ((4 - 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2)
/E^(I*ArcTan[2 + I]) - ((2 - 4*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2
]]^2)/E^ArcTanh[1 + 2*I] + (10*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1
 + 2*I] - (5 - 5*I)*Pi*Log[1 + E^((-2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])]
+ 10*ArcTan[2 + I]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sq
rt[c*x^2]]))] - 10*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTa
n[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + (10*I)*ArcTan[1 - Sqrt[2]*S
qrt[c*x^2]]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1
+ 2*I])] - 10*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x
```

```
^2]] - 2*ArcTanh[1 + 2*I])] + (5 - 5*I)*Pi*Log[1/Sqrt[2 + 2*c*x^2 - 2*Sqrt[
2]*Sqrt[c*x^2]]] - 10*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqr
t[2]*Sqrt[c*x^2]]]] + 10*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 - Sqrt[2]*Sqrt[c
*x^2]] + I*ArcTanh[1 + 2*I]]] + (5*I)*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] +
 ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 5*PolyLog[2, E^((2*I)*ArcTan[1 - Sqrt
[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 - Sqrt[2]*Sq
rt[c*x^2]]] - 2*Sin[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]))/(1 + c^2*x^4) - (S
qrt[c*x^2]*(1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2)^(3/2)*(2*(-5*ArcTan[2 + I]*Arc
Tan[1 + Sqrt[2]*Sqrt[c*x^2]] + 4*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2 + ((1 +
2*I)*Sqrt[1 + I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) + (
(1 - 2*I)*Sqrt[1 - I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I]
 - (5*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] + (5*I)*(-ArcTan[
2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 - E^((2*I)*(-ArcTan[2 + I]
+ ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] + 5*((-I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^
2]] + ArcTanh[1 + 2*I])*Log[1 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] -
2*ArcTanh[1 + 2*I])] + (5*I)*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[
1 + Sqrt[2]*Sqrt[c*x^2]]]] - 5*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 + Sqrt[2]*
Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]]) + 5*PolyLog[2, E^((2*I)*(-ArcTan[2 + I
] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] - 5*PolyLog[2, E^((2*I)*ArcTan[1 + S
qrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 + Sqrt[2]
*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]))/(20*Sqrt[2]*(-1
 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1
 + (1 + Sqrt[2]*Sqrt[c*x^2])^2] - (1 + Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 + S
qrt[2]*Sqrt[c*x^2])^2])) - ((1/40 + I/40)*c*x^2*(1 + (1 + Sqrt[2]*Sqrt[c*x^
2])^2)*((5 + 5*I)*Pi*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] + 10*ArcTan[2 + I]*Arc
Tan[1 + Sqrt[2]*Sqrt[c*x^2]] + (4 - 4*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2
- ((2 + 4*I)*Sqrt[1 + I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 +
 I]) + ((4 + 2*I)*Sqrt[1 - I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[
1 + 2*I] + 10*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] + (5 - 5*I)*
Pi*Log[1 + E^((-2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])] + (10*I)*ArcTan[2 +
I]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] -
(10*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + A
rcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] + 10*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[
1 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (10*I
)*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*Arc
Tanh[1 + 2*I])] - (5 - 5*I)*Pi*Log[1/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2]]
 - (10*I)*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 + Sqrt[2]*Sqrt[c*
x^2]]]] - (10*I)*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] +
 I*ArcTanh[1 + 2*I]]] - 5*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 +
Sqrt[2]*Sqrt[c*x^2]]))] - (5*I)*PolyLog[2, E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt
[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2
]]] - 2*Sin[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]))/((-1 - c*x^2 + Sqrt[2]*Sqr
t[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c
*x^2])^2] - (1 + Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2]
)^2) + ((1/80 + I/80)*(1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2)^2*((-5 - 5*I)*Pi*Ar
```

```
cTan[1 + Sqrt[2]*Sqrt[c*x^2]] - (10*I)*ArcTan[2 + I]*ArcTan[1 + Sqrt[2]*Sqr
t[c*x^2]] + (8 - 8*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2 - ((4 - 2*I)*Sqrt[1
 + I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) - ((2 - 4*I)*S
qrt[1 - I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I] + (10*I)*A
rcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] - (5 - 5*I)*Pi*Log[1 + E^((
-2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])] + 10*ArcTan[2 + I]*Log[1 - E^((2*I)
*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] - 10*ArcTan[1 + Sqrt[
2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[
c*x^2]]))] + (10*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*ArcTan
[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] - 10*ArcTanh[1 + 2*I]*Log[
1 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5 -
5*I)*Pi*Log[1/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2]] - 10*ArcTan[2 + I]*Log
[-Sin[ArcTan[2 + I] - ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]] + 10*ArcTanh[1 + 2*
I]*Log[Sin[ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]] + (5*I)*P
olyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] + 5
*PolyLog[2, E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])]
)*(3 + 2*Cos[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 + Sqrt[2
]*Sqrt[c*x^2]]]))/((-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*
Sqrt[c*x^2])))/(2*Sqrt[2]))))/(2*c*x)
```

Maple [F]

$$\int (ex+d)\left(a+b\arctan\left(c\,x^{2}\right)\right)^{2}dx$$

```
[In] int((e*x+d)*(a+b*arctan(c*x^2))^2,x)
[Out] int((e*x+d)*(a+b*arctan(c*x^2))^2,x)
```

Fricas [F]

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int (ex+d)\left(b\arctan\left(cx^2\right)+a\right)^2\,dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
[Out] integral(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctan(c*x^2)^2 + 2*(a*b*e*x +
 a*b*d)*arctan(c*x^2), x)
```

Sympy [F]

$$\int (d+ex) \left(a+b \arctan \left(cx^2\right)\right)^2 \, dx = \int \left(a+b \operatorname{atan}\left(cx^2\right)\right)^2 (d+ex) \, dx$$

```
[In] integrate((e*x+d)*(a+b*atan(c*x**2))**2,x)
[Out] Integral((a + b*atan(c*x**2))**2*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex) \left(a+b \arctan \left(cx^2\right)\right)^2 \, dx = \int (ex+d)\left(b \arctan \left(cx^2\right)+a\right)^2 dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
[Out] 12*b^2*c^2*e*integrate(1/16*x^5*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + b^2*c^2
*e*integrate(1/16*x^5*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 12*b^2*c^2*d*i
ntegrate(1/16*x^4*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 4*b^2*c^2*e*integrate
(1/16*x^5*log(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + b^2*c^2*d*integrate(1/16*x^4
*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 8*b^2*c^2*d*integrate(1/16*x^4*log(
c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 1/2*a^2*e*x^2 + 1/8*b^2*e*arctan(c*x^2)^3/
c - 8*b^2*c*e*integrate(1/16*x^3*arctan(c*x^2)/(c^2*x^4 + 1), x) - 16*b^2*c
*d*integrate(1/16*x^2*arctan(c*x^2)/(c^2*x^4 + 1), x) - 1/2*(c*(2*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) + 2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) - sqrt(2)*log(c
*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)
*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*a*b*d + a^2*d*x + b^2*e*integrate(1/1
6*x*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 12*b^2*d*integrate(1/16*arctan(c
*x^2)^2/(c^2*x^4 + 1), x) + b^2*d*integrate(1/16*log(c^2*x^4 + 1)^2/(c^2*x^
4 + 1), x) + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*a*b*e/c + 1/8*(
b^2*e*x^2 + 2*b^2*d*x)*arctan(c*x^2)^2 - 1/32*(b^2*e*x^2 + 2*b^2*d*x)*log(c
^2*x^4 + 1)^2
```

Giac [F]

$$\int (d+ex) \left(a+b \arctan \left(cx^2\right)\right)^2 \, dx = \int (ex+d)\left(b \arctan \left(cx^2\right)+a\right)^2 dx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*arctan(c*x^2) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,(d+e\,x)\,dx$$

```
[In] int((a + b*atan(c*x^2))^2*(d + e*x),x)
[Out] int((a + b*atan(c*x^2))^2*(d + e*x), x)
```

3.26 $\int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx = \text{Int}\left(\frac{(a+b\arctan(cx^2))^2}{d+ex},x\right)$$

```
[Out] Unintegrable((a+b*arctan(c*x^2))^2/(e*x+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx = \int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2/(d + e*x),x]
[Out] Defer[Int][(a + b*ArcTan[c*x^2])^2/(d + e*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx$$

Mathematica [N/A]

Not integrable

Time = 51.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx = \int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/(d + e*x),x]
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/(d + e*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arctan(c\,x^2))^2}{ex+d}dx$$

```
[In] int((a+b*arctan(c*x^2))^2/(e*x+d),x)
[Out] int((a+b*arctan(c*x^2))^2/(e*x+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(c*x**2))**2/(e*x+d),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{(a + b \arctan(cx^2))^2}{d + ex} \, dx = \int \frac{(b \arctan(cx^2) + a)^2}{ex + d} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2))/
(e*x + d), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \arctan(cx^2))^2}{d + ex} \, dx = \int \frac{(b \arctan(cx^2) + a)^2}{ex + d} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/(e*x + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \arctan(cx^2))^2}{d + ex} \, dx = \int \frac{(a + b \operatorname{atan}(c x^2))^2}{d + e x} \, dx$$

```
[In] int((a + b*atan(c*x^2))^2/(d + e*x),x)
```

```
[Out] int((a + b*atan(c*x^2))^2/(d + e*x), x)
```

3.27 $\int \frac{(a+b\arctan(cx^2))^2}{(d+ex)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\arctan(cx^2))^2}{(d+ex)^2} dx = \text{Int}\left(\frac{(a+b\arctan(cx^2))^2}{(d+ex)^2}, x\right)$$

```
[Out] Unintegrable((a+b*arctan(c*x^2))^2/(e*x+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arctan(cx^2))^2}{(d+ex)^2} dx = \int \frac{(a+b\arctan(cx^2))^2}{(d+ex)^2} dx$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2/(d + e*x)^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTan[c*x^2])^2/(d + e*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\arctan(cx^2))^2}{(d+ex)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 108.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\arctan(cx^2))^2}{(d+ex)^2}\,dx=\int \frac{(a+b\arctan(cx^2))^2}{(d+ex)^2}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/(d + e*x)^2,x]
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/(d + e*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arctan(c\,x^2))^2}{(ex+d)^2}dx$$

```
[In] int((a+b*arctan(c*x^2))^2/(e*x+d)^2,x)
[Out] int((a+b*arctan(c*x^2))^2/(e*x+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{(a+b\arctan(cx^2))^2}{(d+ex)^2}\,dx=\int \frac{(b\arctan(cx^2)+a)^2}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/(e^2*x^2 + 2*d*e
*x + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^2))^2}{(d + ex)^2} \, dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(c*x**2))**2/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\arctan(cx^2))^2}{(d + ex)^2} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d)^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b\arctan(cx^2))^2}{(d + ex)^2} \, dx = \int \frac{(b\arctan(cx^2) + a)^2}{(ex + d)^2} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/(e*x + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b\arctan(cx^2))^2}{(d + ex)^2} \, dx = \int \frac{(a + b\,\mathrm{atan}(c\,x^2))^2}{(d + e\,x)^2} \, dx$$

```
[In] int((a + b*atan(c*x^2))^2/(d + e*x)^2,x)
[Out] int((a + b*atan(c*x^2))^2/(d + e*x)^2, x)
```

3.28 $\int (d+ex)^2 \left(a+b\arctan\left(cx^3\right)\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 315

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^3\right)\right) \, dx = -\frac{bde\arctan\left(\sqrt[3]{c}x\right)}{c^{2/3}} - \frac{bd^3\arctan\left(cx^3\right)}{3e} + \frac{(d+ex)^3\left(a+b\arctan\left(cx^3\right)\right)}{3e} + \frac{bde\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bde\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{2c^{2/3}} + \frac{\sqrt{3}bd^2\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + \frac{bd^2\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{\sqrt{3}bde\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4c^{2/3}} + \frac{\sqrt{3}bde\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4c^{2/3}} - \frac{bd^2\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}} - \frac{be^2\log\left(1+c^2x^6\right)}{6c}$$

```
[Out] -b*d*e*arctan(c^(1/3)*x)/c^(2/3)-1/3*b*d^3*arctan(c*x^3)/e+1/3*(e*x+d)^3*(a
+b*arctan(c*x^3))/e-1/2*b*d*e*arctan(2*c^(1/3)*x-3^(1/2))/c^(2/3)-1/2*b*d*e
*arctan(2*c^(1/3)*x+3^(1/2))/c^(2/3)+1/2*b*d^2*ln(1+c^(2/3)*x^2)/c^(1/3)-1/
4*b*d^2*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)/c^(1/3)-1/6*b*e^2*ln(c^2*x^6+1)/c+1/2
*b*d^2*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)/c^(1/3)-1/4*b*d*e*ln(1
+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/4*b*d*e*ln(1+c^(2/3)*x^2+
c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4980, 1845, 281, 298, 31, 648, 631, 210, 642, 301, 632, 209, 1483, 649, 266}

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^3\right)\right)\,dx = \frac{(d+ex)^3\left(a+b\arctan\left(cx^3\right)\right)}{3e} + \frac{\sqrt{3}bd^2\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{bde\arctan\left(\sqrt[3]{c}x\right)}{c^{2/3}} + \frac{bde\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bde\arctan\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{2c^{2/3}} - \frac{bd^3\arctan\left(cx^3\right)}{3e} + \frac{bd^2\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}} - \frac{bd^2\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}} - \frac{\sqrt{3}bde\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{4c^{2/3}} + \frac{\sqrt{3}bde\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{4c^{2/3}} - \frac{be^2\log\left(c^2x^6+1\right)}{6c}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -((b*d*e*ArcTan[c^(1/3)*x])/c^(2/3)) - (b*d^3*ArcTan[c*x^3])/(3*e) + ((d +
e*x)^3*(a + b*ArcTan[c*x^3]))/(3*e) + (b*d*e*ArcTan[Sqrt[3] - 2*c^(1/3)*x])
/(2*c^(2/3)) - (b*d*e*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(2*c^(2/3)) + (Sqrt[3]
*b*d^2*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + (b*d^2*Log[1 + c^
(2/3)*x^2])/(2*c^(1/3)) - (Sqrt[3]*b*d*e*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3
)*x^2])/(4*c^(2/3)) + (Sqrt[3]*b*d*e*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^
2])/(4*c^(2/3)) - (b*d^2*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(4*c^(1/3)) -
(b*e^2*Log[1 + c^2*x^6])/(6*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_) + (e_.)*(x_))^(m_.), x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(e*(m + 1))), x] - D
ist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 + c^2*x^(2*n))
), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^3\,(a+b\arctan(cx^3))}{3e} - \frac{(bc)\int \frac{x^2(d+ex)^3}{1+c^2x^6}\,dx}{e}$$

$$= \frac{(d+ex)^3(a+b\arctan(cx^3))}{3e} - \frac{(bc)\int\left(\frac{3d^2ex^3}{1+c^2x^6}+\frac{3de^2x^4}{1+c^2x^6}+\frac{x^2(d^3+e^3x^3)}{1+c^2x^6}\right)dx}{e}$$

$$= \frac{(d+ex)^3(a+b\arctan(cx^3))}{3e} - (3bcd^2)\int\frac{x^3}{1+c^2x^6}\,dx$$

$$- \frac{(bc)\int\frac{x^2(d^3+e^3x^3)}{1+c^2x^6}\,dx}{e} - (3bcde)\int\frac{x^4}{1+c^2x^6}\,dx$$

$$= \frac{(d+ex)^3(a+b\arctan(cx^3))}{3e} - \frac{1}{2}(3bcd^2)\,\text{Subst}\left(\int\frac{x}{1+c^2x^3}\,dx, x, x^2\right)$$

$$- \frac{(bc)\text{Subst}\left(\int\frac{d^3+e^3x}{1+c^2x^2}\,dx, x, x^3\right)}{3e} - \frac{(bde)\int\frac{1}{1+c^{2/3}x^2}\,dx}{\sqrt[3]{c}}$$

$$- \frac{(bde)\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{\sqrt[3]{c}} - \frac{(bde)\int\frac{-\frac{1}{2}-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{\sqrt[3]{c}}$$

$$= -\frac{bde\arctan\left(\sqrt[3]{c}x\right)}{c^{2/3}} + \frac{(d+ex)^3(a+b\arctan(cx^3))}{3e}$$

$$+ \frac{1}{2}\left(b\sqrt[3]{c}d^2\right)\text{Subst}\left(\int\frac{1}{1+c^{2/3}x}\,dx, x, x^2\right)$$

$$- \frac{1}{2}\left(b\sqrt[3]{c}d^2\right)\text{Subst}\left(\int\frac{1+c^{2/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right) - \frac{(bcd^3)\,\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx, x, x^3\right)}{3e} - \frac{\left(\sqrt{3}bde\right)\cdots}{\cdots}$$

$$= -\frac{bde\arctan\left(\sqrt[3]{c}x\right)}{c^{2/3}} - \frac{bd^3\arctan(cx^3)}{3e}$$

$$+ \frac{(d+ex)^3(a+b\arctan(cx^3))}{3e} + \frac{bd^2\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}$$

$$- \frac{\sqrt{3}bde\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4c^{2/3}} + \frac{\sqrt{3}bde\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4c^{2/3}}$$

$$- \frac{be^2\log(1+c^2x^6)}{6c} - \frac{(bd^2)\,\text{Subst}\left(\int\frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right)}{4\sqrt[3]{c}}$$

$$- \frac{1}{4}\left(3b\sqrt[3]{c}d^2\right)\text{Subst}\left(\int\frac{1}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right) - \frac{(bde)\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{2\sqrt{3}c^{2/3}} + \cdots$$

$$= -\frac{bde\arctan\left(\sqrt[3]{c}x\right)}{c^{2/3}} - \frac{bd^3\arctan(cx^3)}{3e} + \frac{(d+ex)^3(a+b\arctan(cx^3))}{3e} + \frac{bde\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bde\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{2c^{2/3}} + \frac{bd^2\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{\sqrt{3}bde\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4c^{2/3}} + \frac{\sqrt{3}bde\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4c^{2/3}} - \frac{bd^2\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}} - \frac{be^2\log(1+c^2x^6)}{6c} - \frac{(3bd^2)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-2c^{2/3}x^2\right)}{2\sqrt[3]{c}}$$

$$= -\frac{bde\arctan\left(\sqrt[3]{c}x\right)}{c^{2/3}} - \frac{bd^3\arctan(cx^3)}{3e} + \frac{(d+ex)^3(a+b\arctan(cx^3))}{3e} + \frac{bde\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bde\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{2c^{2/3}} + \frac{\sqrt{3}bd^2\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + \frac{bd^2\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{\sqrt{3}bde\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4c^{2/3}} + \frac{\sqrt{3}bde\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4c^{2/3}} - \frac{bd^2\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}} - \frac{be^2\log(1+c^2x^6)}{6c}$$

Mathematica [A] (verified)

Time = 106.47 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.94

$$\int (d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)\,dx = \frac{12acd^2x + 12acdex^2 + 4ace^2x^3 - 12b\sqrt[3]{c}de\arctan\left(\sqrt[3]{c}x\right) + 4bcx(3d^2+3dex+e^2x^2)\arctan(cx^3) + 6b\sqrt[3]{c}d$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (12*a*c*d^2*x + 12*a*c*d*e*x^2 + 4*a*c*e^2*x^3 - 12*b*c^(1/3)*d*e*ArcTan[c^
(1/3)*x] + 4*b*c*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTan[c*x^3] + 6*b*c^(1/3)*
d*(Sqrt[3]*c^(1/3)*d + e)*ArcTan[Sqrt[3] - 2*c^(1/3)*x] + 6*b*c^(1/3)*d*(Sq
rt[3]*c^(1/3)*d - e)*ArcTan[Sqrt[3] + 2*c^(1/3)*x] + 6*b*c^(2/3)*d^2*Log[1
+ c^(2/3)*x^2] - 3*b*c^(1/3)*d*(c^(1/3)*d + Sqrt[3]*e)*Log[1 - Sqrt[3]*c^(1
/3)*x + c^(2/3)*x^2] - 3*b*c^(1/3)*d*(c^(1/3)*d - Sqrt[3]*e)*Log[1 + Sqrt[3
]*c^(1/3)*x + c^(2/3)*x^2] - 2*b*e^2*Log[1 + c^2*x^6])/(12*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(244) = 488$.

Time = 33.36 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.60

method	result
default	$\frac{a(ex+d)^3}{3e} + b\left(\frac{e^2 \arctan(c x^3) x^3}{3} + e \arctan(c x^3) d x^2 + \arctan(c x^3) d^2 x + \frac{\arctan(c x^3) d^3}{3e} - \frac{c\left(-\frac{\ln\left(x^2+\sqrt{3}\right.}{}\right.}{}\right.$
parts	$\frac{a(ex+d)^3}{3e} + b\left(\frac{e^2 \arctan(c x^3) x^3}{3} + e \arctan(c x^3) d x^2 + \arctan(c x^3) d^2 x + \frac{\arctan(c x^3) d^3}{3e} - \frac{c\left(-\frac{\ln\left(x^2+\sqrt{3}\right.}{}\right.}{}\right.$

```
[In] int((e*x+d)^2*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(e*x+d)^3/e+b*(1/3*e^2*arctan(c*x^3)*x^3+e*arctan(c*x^3)*d*x^2+arctan
(c*x^3)*d^2*x+1/3/e*arctan(c*x^3)*d^3-c/e*(-1/4*ln(x^2+3^(1/2)*(1/c^2)^(1/6
)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*d*e^2+1/4*ln(x^2+3^(1/2)*(1/c^2)^(
1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3)*d^2*e+1/6/c^2*ln(x^2+3^(1/2)*(1/c^2)^(1
/6)*x+(1/c^2)^(1/3))*e^3+1/2/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(
1/2))*d*e^2-1/2*(1/c^2)^(2/3)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*3^(1/2)*d^2
*e+1/3*(1/c^2)^(1/2)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*d^3+1/4*ln(x^2-3^(1/
2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*d*e^2+1/4*ln(x^2-3^
(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3)*d^2*e+1/6/c^2*ln(x^2-3^(
1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*e^3+1/2/c^2/(1/c^2)^(1/6)*arctan(2*x/(1
/c^2)^(1/6)-3^(1/2))*d*e^2+1/2*(1/c^2)^(2/3)*arctan(2*x/(1/c^2)^(1/6)-3^(1/
2))*3^(1/2)*d^2*e+1/3*(1/c^2)^(1/2)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*d^3-1
/2*ln(x^2+(1/c^2)^(1/3))*(1/c^2)^(2/3)*d^2*e+1/6/c^2*ln(x^2+(1/c^2)^(1/3))*
e^3-1/3*(1/c^2)^(1/2)*arctan(x/(1/c^2)^(1/6))*d^3+1/c^2/(1/c^2)^(1/6)*arcta
n(x/(1/c^2)^(1/6))*d*e^2))
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex)^2 \left(a+b\arctan \left(cx^3\right)\right) \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^3)),x, algorithm="fricas")
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [A] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.48

$$\begin{aligned}
&\int (d+ex)^2 \left(a+b\arctan \left(cx^3\right)\right) \, dx \\
&= ad^2x + adex^2 + \frac{ae^2x^3}{3} - 3bcd^2 \operatorname{RootSum}\left(216t^3c^4+1, \left(t \mapsto t\log\left(36t^2c^2+x^2\right)\right)\right) \\
&\quad - 3bcde \operatorname{RootSum}\left(46656t^6c^{10}+1, \left(t \mapsto t\log\left(7776t^5c^8+x\right)\right)\right) + bd^2x \operatorname{atan}\left(cx^3\right) \\
&\quad + bdex^2 \operatorname{atan}\left(cx^3\right) + be^2 \left(\begin{cases} 0 & \text{for}\ c=0 \\ \frac{x^3 \operatorname{atan}\left(cx^3\right)}{3} - \frac{\log\left(c^2x^6+1\right)}{6c} & \text{otherwise} \end{cases}\right)
\end{aligned}$$

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x**3)),x)
[Out] a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 - 3*b*c*d**2*RootSum(216*_t**3*c**4 +
 1, Lambda(_t, _t*log(36*_t**2*c**2 + x**2))) - 3*b*c*d*e*RootSum(46656*_t*
*6*c**10 + 1, Lambda(_t, _t*log(7776*_t**5*c**8 + x))) + b*d**2*x*atan(c*x*
*3) + b*d*e*x**2*atan(c*x**3) + b*e**2*Piecewise((0, Eq(c, 0)), (x**3*atan(
c*x**3)/3 - log(c**2*x**6 + 1)/(6*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.89

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^3\right)\right)\,dx = \frac{1}{3}ae^2x^3 + adex^2 - \frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}} + \frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}} - \frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right) - 4x\arctan\left(cx^3\right)\right. + \frac{1}{4}\left(4x^2\arctan\left(cx^3\right) + c\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}} - \frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}} - \frac{4\arctan\left(c^{\frac{1}{3}}\right.}{c^{\frac{5}{3}}}\right.\right. + ad^2x + \frac{(2cx^3\arctan(cx^3) - \log(c^2x^6+1))be^2}{6c}$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 - 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)
*x^2 - c^(2/3))/c^(2/3))/c^(4/3) + log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(4/
3) - 2*log((c^(2/3)*x^2 + 1)/c^(2/3))/c^(4/3)) - 4*x*arctan(c*x^3))*b*d^2 +
 1/4*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x
+ 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - 4
*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/
3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3)))*b
*d*e + a*d^2*x + 1/6*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b*e^2/c
```

Giac [A] (verification not implemented)

none

Time = 9.73 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.99

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$= \frac{1}{3}be^2x^3\arctan\left(cx^3\right)+\frac{1}{3}ae^2x^3+bdex^2\arctan\left(cx^3\right)+adex^2+bd^2x\arctan\left(cx^3\right)$$
$$+ad^2x-\frac{bcde\arctan\left(x|c|^{\frac{1}{3}}\right)}{|c|^{\frac{5}{3}}}+\frac{\left(\sqrt{3}bcd^2|c|^{\frac{1}{3}}-bcde\right)\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{2|c|^{\frac{5}{3}}}$$
$$-\frac{\left(\sqrt{3}bcd^2|c|^{\frac{1}{3}}+bcde\right)\arctan\left(\left(2x-\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{2|c|^{\frac{5}{3}}}$$
$$+\frac{\left(3\sqrt{3}bcde|c|^{\frac{1}{3}}-3bcd^2|c|^{\frac{2}{3}}-2bce^2\right)\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{12c^2}$$
$$-\frac{\left(3\sqrt{3}bcde|c|^{\frac{1}{3}}+3bcd^2|c|^{\frac{2}{3}}+2bce^2\right)\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{12c^2}$$
$$+\frac{\left(3bcd^2|c|^{\frac{2}{3}}-bce^2\right)\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{6c^2}$$

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/3*b*e^2*x^3*arctan(c*x^3) + 1/3*a*e^2*x^3 + b*d*e*x^2*arctan(c*x^3) + a*d
*e*x^2 + b*d^2*x*arctan(c*x^3) + a*d^2*x - b*c*d*e*arctan(x*abs(c)^(1/3))/a
bs(c)^(5/3) + 1/2*(sqrt(3)*b*c*d^2*abs(c)^(1/3) - b*c*d*e)*arctan((2*x + sq
rt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(5/3) - 1/2*(sqrt(3)*b*c*d^2*abs(c
)^(1/3) + b*c*d*e)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)
^(5/3) + 1/12*(3*sqrt(3)*b*c*d*e*abs(c)^(1/3) - 3*b*c*d^2*abs(c)^(2/3) - 2*
b*c*e^2)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/c^2 - 1/12*(3*s
qrt(3)*b*c*d*e*abs(c)^(1/3) + 3*b*c*d^2*abs(c)^(2/3) + 2*b*c*e^2)*log(x^2 -
 sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/c^2 + 1/6*(3*b*c*d^2*abs(c)^(2/3)
 - b*c*e^2)*log(x^2 + 1/abs(c)^(2/3))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 988, normalized size of antiderivative = 3.14

$$\int (d+ex)^2 \left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$= \mathrm{atan}\left(c\,x^3\right)\,\left(b\,d^2\,x+b\,d\,e\,x^2+\frac{b\,e^2\,x^3}{3}\right)+\left(\sum_{k=1}^{6}\ln\left(x\,\left(6\,b^5\,c^7\,d^2\,e^8-162\,b^5\,c^9\,d^8\,e^2\right)\right.\right.$$
$$+\mathrm{root}\left(46656\,a^6\,c^6+46656\,a^5\,b\,c^5\,e^2+19440\,a^4\,b^2\,c^4\,e^4+4320\,a^3\,b^3\,c^3\,e^6-11664\,a^3\,b^3\,c^5\,d^6+20412\,a^2\,b^4\,c^4\,d^6\right.$$
$$\left.-243\,b^5\,c^9\,d^9\,e+9\,b^5\,c^7\,d^3\,e^7\right)\,\mathrm{root}\left(46656\,a^6\,c^6+46656\,a^5\,b\,c^5\,e^2+19440\,a^4\,b^2\,c^4\,e^4\right.$$
$$+4320\,a^3\,b^3\,c^3\,e^6-11664\,a^3\,b^3\,c^5\,d^6+20412\,a^2\,b^4\,c^4\,d^6\,e^2+540\,a^2\,b^4\,c^2\,e^8-972\,a\,b^5\,c^3\,d^6\,e^4$$
$$\left.\left.+36\,a\,b^5\,c\,e^{10}-54\,b^6\,c^2\,d^6\,e^6+729\,b^6\,c^4\,d^{12}+b^6\,e^{12},a,k\right)\right)+\frac{a\,e^2\,x^3}{3}+a\,d^2\,x+a\,d\,e\,x^2$$

```
[In] int((a + b*atan(c*x^3))*(d + e*x)^2,x)
```

```
[Out] atan(c*x^3)*((b*e^2*x^3)/3 + b*d^2*x + b*d*e*x^2) + symsum(log(x*(6*b^5*c^7
*d^2*e^8 - 162*b^5*c^9*d^8*e^2) + root(46656*a^6*c^6 + 46656*a^5*b*c^5*e^2
+ 19440*a^4*b^2*c^4*e^4 + 4320*a^3*b^3*c^3*e^6 - 11664*a^3*b^3*c^5*d^6 + 20
412*a^2*b^4*c^4*d^6*e^2 + 540*a^2*b^4*c^2*e^8 - 972*a*b^5*c^3*d^6*e^4 + 36*
a*b^5*c*e^10 - 54*b^6*c^2*d^6*e^6 + 729*b^6*c^4*d^12 + b^6*e^12, a, k)*(x*(
486*b^4*c^10*d^8 + 90*b^4*c^8*d^2*e^6) - root(46656*a^6*c^6 + 46656*a^5*b*c
^5*e^2 + 19440*a^4*b^2*c^4*e^4 + 4320*a^3*b^3*c^3*e^6 - 11664*a^3*b^3*c^5*d
^6 + 20412*a^2*b^4*c^4*d^6*e^2 + 540*a^2*b^4*c^2*e^8 - 972*a*b^5*c^3*d^6*e^
4 + 36*a*b^5*c*e^10 - 54*b^6*c^2*d^6*e^6 + 729*b^6*c^4*d^12 + b^6*e^12, a,
k)*(root(46656*a^6*c^6 + 46656*a^5*b*c^5*e^2 + 19440*a^4*b^2*c^4*e^4 + 4320
*a^3*b^3*c^3*e^6 - 11664*a^3*b^3*c^5*d^6 + 20412*a^2*b^4*c^4*d^6*e^2 + 540*
a^2*b^4*c^2*e^8 - 972*a*b^5*c^3*d^6*e^4 + 36*a*b^5*c*e^10 - 54*b^6*c^2*d^6*
e^6 + 729*b^6*c^4*d^12 + b^6*e^12, a, k)*(3888*b^2*c^10*d^3*e + 3888*root(4
6656*a^6*c^6 + 46656*a^5*b*c^5*e^2 + 19440*a^4*b^2*c^4*e^4 + 4320*a^3*b^3*c
^3*e^6 - 11664*a^3*b^3*c^5*d^6 + 20412*a^2*b^4*c^4*d^6*e^2 + 540*a^2*b^4*c^
2*e^8 - 972*a*b^5*c^3*d^6*e^4 + 36*a*b^5*c*e^10 - 54*b^6*c^2*d^6*e^6 + 729*
b^6*c^4*d^12 + b^6*e^12, a, k)*b*c^11*d^2*x + 648*b^2*c^10*d^2*e^2*x) + 972
*b^3*c^9*d^3*e^3 - 324*b^3*c^9*d^2*e^4*x)) - 243*b^5*c^9*d^9*e + 9*b^5*c^7*
d^3*e^7)*root(46656*a^6*c^6 + 46656*a^5*b*c^5*e^2 + 19440*a^4*b^2*c^4*e^4 +
 4320*a^3*b^3*c^3*e^6 - 11664*a^3*b^3*c^5*d^6 + 20412*a^2*b^4*c^4*d^6*e^2 +
 540*a^2*b^4*c^2*e^8 - 972*a*b^5*c^3*d^6*e^4 + 36*a*b^5*c*e^10 - 54*b^6*c^2
*d^6*e^6 + 729*b^6*c^4*d^12 + b^6*e^12, a, k), k, 1, 6) + (a*e^2*x^3)/3 + a
*d^2*x + a*d*e*x^2
```

3.29 $\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 285

$$\begin{aligned}\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx =& -\frac{be\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}-\frac{bd^2\arctan\left(cx^3\right)}{2e}\\&+\frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e}\\&+\frac{be\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{be\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}}\\&+\frac{\sqrt{3}bd\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+\frac{bd\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}\\&-\frac{\sqrt{3}be\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}\\&+\frac{\sqrt{3}be\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}\\&-\frac{bd\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}\end{aligned}$$

```
[Out] -1/2*b*e*arctan(c^(1/3)*x)/c^(2/3)-1/2*b*d^2*arctan(c*x^3)/e+1/2*(e*x+d)^2*
(a+b*arctan(c*x^3))/e-1/4*b*e*arctan(2*c^(1/3)*x-3^(1/2))/c^(2/3)-1/4*b*e*a
rctan(2*c^(1/3)*x+3^(1/2))/c^(2/3)+1/2*b*d*ln(1+c^(2/3)*x^2)/c^(1/3)-1/4*b*
d*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)/c^(1/3)+1/2*b*d*arctan(1/3*(1-2*c^(2/3)*x^2
)*3^(1/2))*3^(1/2)/c^(1/3)-1/8*b*e*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1
/2)/c^(2/3)+1/8*b*e*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4980, 1845, 281, 209, 298, 31, 648, 631, 210, 642, 301, 632}

$$\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e}+\frac{\sqrt{3}bd\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{be\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{be\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{be\arctan\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{4c^{2/3}}-\frac{bd^2\arctan\left(cx^3\right)}{2e}+\frac{bd\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{bd\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}-\frac{\sqrt{3}be\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}be\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}$$

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/2*(b*e*ArcTan[c^(1/3)*x])/c^(2/3) - (b*d^2*ArcTan[c*x^3])/(2*e) + ((d +
e*x)^2*(a + b*ArcTan[c*x^3]))/(2*e) + (b*e*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(
4*c^(2/3)) - (b*e*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(4*c^(2/3)) + (Sqrt[3]*b*d
*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + (b*d*Log[1 + c^(2/3)*x^
2])/(2*c^(1/3)) - (Sqrt[3]*b*e*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8
*c^(2/3)) + (Sqrt[3]*b*e*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/
3)) - (b*d*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_) + (e_.)*(x_))^(m_.), x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(e*(m + 1))), x] - D
ist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 + c^2*x^(2*n))
), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e}-\frac{(3bc)\int\frac{x^2(d+ex)^2}{1+c^2x^6}\,dx}{2e}\\
&= \frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e}-\frac{(3bc)\int\left(\frac{d^2x^2}{1+c^2x^6}+\frac{2dex^3}{1+c^2x^6}+\frac{e^2x^4}{1+c^2x^6}\right)dx}{2e}\\
&= \frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e}-(3bcd)\int\frac{x^3}{1+c^2x^6}\,dx\\
&\quad-\frac{\left(3bcd^2\right)\int\frac{x^2}{1+c^2x^6}\,dx}{2e}-\frac{1}{2}(3bce)\int\frac{x^4}{1+c^2x^6}\,dx\\
&= \frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e}-\frac{1}{2}(3bcd)\text{Subst}\left(\int\frac{x}{1+c^2x^3}\,dx,x,x^2\right)\\
&\quad-\frac{\left(bcd^2\right)\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{2e}-\frac{(be)\int\frac{1}{1+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}\\
&\quad-\frac{(be)\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{(be)\int\frac{-\frac{1}{2}-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}\\
&= -\frac{be\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}-\frac{bd^2\arctan\left(cx^3\right)}{2e}+\frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e}\\
&\quad+\frac{1}{2}\left(b\sqrt[3]{c}d\right)\text{Subst}\left(\int\frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)\\
&\quad-\frac{1}{2}\left(b\sqrt[3]{c}d\right)\text{Subst}\left(\int\frac{1+c^{2/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)-\frac{\left(\sqrt{3}be\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{2/3}}+\frac{\left(\sqrt{3}be\right)\int\frac{\sqrt{\cdots}}{1+\cdots}}{8\cdots}
\end{aligned}
$$

$$= -\frac{be\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bd^2\arctan\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e} + \frac{bd\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{\sqrt{3}be\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}} + \frac{\sqrt{3}be\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}} - \frac{(bd)\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)}{4\sqrt[3]{c}} - \frac{1}{4}\left(3b\sqrt[3]{c}d\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right) - \frac{(be)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4\sqrt{3}c^{2/3}} + \frac{(be)}{}$$

$$= -\frac{be\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bd^2\arctan\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e} + \frac{be\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}} - \frac{be\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}} + \frac{bd\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{\sqrt{3}be\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}} + \frac{\sqrt{3}be\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}} - \frac{bd\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}} - \frac{(3bd)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-2c^{2/3}x^2\right)}{2\sqrt[3]{c}}$$

$$= -\frac{be\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bd^2\arctan\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a+b\arctan\left(cx^3\right)\right)}{2e} + \frac{be\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}} - \frac{be\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}} + \frac{\sqrt{3}bd\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + \frac{bd\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{\sqrt{3}be\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}} + \frac{\sqrt{3}be\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}} - \frac{bd\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.09

$$\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx = adx + \frac{1}{2}aex^2 - \frac{be\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}} + bdx\arctan\left(cx^3\right) + \frac{1}{2}bex^2\arctan\left(cx^3\right) + \frac{be\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}} - \frac{be\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}} - \frac{\sqrt{3}be\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}} + \frac{\sqrt{3}be\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}} - \frac{bd\left(-2\sqrt{3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)-2\sqrt{3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-2\log\left(1+c^{2/3}x^2\right)+\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}\right.\right.}{4\sqrt[3]{c}}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*e*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + b*d*x*ArcTan[c*
x^3] + (b*e*x^2*ArcTan[c*x^3])/2 + (b*e*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(4*c
^(2/3)) - (b*e*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(4*c^(2/3)) - (Sqrt[3]*b*e*Lo
g[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) + (Sqrt[3]*b*e*Log[1 +
Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*d*(-2*Sqrt[3]*ArcTan[Sqr
t[3] - 2*c^(1/3)*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*c^(1/3)*x] - 2*Log[1 + c
^(2/3)*x^2] + Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2] + Log[1 + Sqrt[3]*c^
(1/3)*x + c^(2/3)*x^2]))/(4*c^(1/3))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.07

method	result
default	$a\left(\frac{1}{2}e\,x^2+dx\right)+b\left(\frac{\arctan(c\,x^3)x^2e}{2}+\arctan\left(c\,x^3\right)dx-\frac{3c\left(-\frac{\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}e}{12}+\frac{\ln\left(x^2+\sqrt{\right.}}{}\right.}{}\right.$
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+b\left(\frac{\arctan(c\,x^3)x^2e}{2}+\arctan\left(c\,x^3\right)dx-\frac{3c\left(-\frac{\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}e}{12}+\frac{\ln\left(x^2+\sqrt{\right.}}{}\right.}{}\right.$

```
[In] int((e*x+d)*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e*x^2+d*x)+b*(1/2*arctan(c*x^3)*x^2*e+arctan(c*x^3)*d*x-3/2*c*(-1/12
*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*e+1/6*
ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3)*d+1/6/c^2/(1/c^
2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*e-1/3*(1/c^2)^(2/3)*arctan(2*x/(
1/c^2)^(1/6)+3^(1/2))*3^(1/2)*d+1/6*c^2*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c
^2)^(1/3))*(1/c^2)^(5/3)*d+1/12*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3
))*3^(1/2)*(1/c^2)^(5/6)*e+1/3*c^2*(1/c^2)^(5/3)*arctan(2*x/(1/c^2)^(1/6)-3
```

```
^(1/2))*3^(1/2)*d+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*e
-1/3*(1/c^2)^(2/3)*d*ln(x^2+(1/c^2)^(1/3))+1/3/c^2*e/(1/c^2)^(1/6)*arctan(x
/(1/c^2)^(1/6))))
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [A] (verification not implemented)

Time = 10.83 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.36

$$\begin{aligned}\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx &= adx+\frac{aex^2}{2}-3bcd\,\mathrm{RootSum}\left(216t^3c^4+1,\left(t\mapsto t\log\left(36t^2c^2+x^2\right)\right)\right)\\&\quad-\frac{3bce\,\mathrm{RootSum}\left(46656t^6c^{10}+1,\left(t\mapsto t\log\left(7776t^5c^8+x\right)\right)\right)}{2}\\&\quad+bdx\,\mathrm{atan}\left(cx^3\right)+\frac{bex^2\,\mathrm{atan}\left(cx^3\right)}{2}\end{aligned}$$

```
[In] integrate((e*x+d)*(a+b*atan(c*x**3)),x)
```

```
[Out] a*d*x + a*e*x**2/2 - 3*b*c*d*RootSum(216*_t**3*c**4 + 1, Lambda(_t, _t*log(
36*_t**2*c**2 + x**2))) - 3*b*c*e*RootSum(46656*_t**6*c**10 + 1, Lambda(_t,
 _t*log(7776*_t**5*c**8 + x)))/2 + b*d*x*atan(c*x**3) + b*e*x**2*atan(c*x**
3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.81

$$\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx = \frac{1}{2}aex^2$$
$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}}-\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)-4x\arctan\left(cx^3\right)\right)$$
$$+\frac{1}{8}\Bigg(4x^2\arctan\left(cx^3\right)+c\Bigg(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(c^{\frac{1}{3}}\right.}{c^{\frac{5}{3}}}$$
$$+adx$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 - 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3)
)/c^(2/3))/c^(4/3) + log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(4/3) - 2*log((c^
(2/3)*x^2 + 1)/c^(2/3))/c^(4/3)) - 4*x*arctan(c*x^3))*b*d + 1/8*(4*x^2*arct
an(c*x^3) + c*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - s
qrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*
x)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*
arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3)))*b*e + a*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.83 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{2}bex^2\arctan\left(cx^3\right)+\frac{1}{2}aex^2+bdx\arctan\left(cx^3\right)+adx+\frac{bcd\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{2\,|c|^{\frac{4}{3}}}-\frac{bce\arctan\left(x|c|^{\frac{1}{3}}\right)}{2\,|c|^{\frac{5}{3}}}+\frac{\left(2\sqrt{3}bcd|c|^{\frac{1}{3}}-bce\right)\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4\,|c|^{\frac{5}{3}}}-\frac{\left(2\sqrt{3}bcd|c|^{\frac{1}{3}}+bce\right)\arctan\left(\left(2\,x-\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4\,|c|^{\frac{5}{3}}}+\frac{\left(\sqrt{3}bce-2\,bcd|c|^{\frac{1}{3}}\right)\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{8\,|c|^{\frac{5}{3}}}-\frac{\left(\sqrt{3}bce+2\,bcd|c|^{\frac{1}{3}}\right)\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{8\,|c|^{\frac{5}{3}}}$$

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/2*b*e*x^2*arctan(c*x^3) + 1/2*a*e*x^2 + b*d*x*arctan(c*x^3) + a*d*x + 1/2
*b*c*d*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3) - 1/2*b*c*e*arctan(x*abs(c)^(
1/3))/abs(c)^(5/3) + 1/4*(2*sqrt(3)*b*c*d*abs(c)^(1/3) - b*c*e)*arctan((2*x
 + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(5/3) - 1/4*(2*sqrt(3)*b*c*d*
abs(c)^(1/3) + b*c*e)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs
(c)^(5/3) + 1/8*(sqrt(3)*b*c*e - 2*b*c*d*abs(c)^(1/3))*log(x^2 + sqrt(3)*x/
abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) - 1/8*(sqrt(3)*b*c*e + 2*b*c*d*
abs(c)^(1/3))*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/
3)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.70

$$\int (d+ex)\left(a+b\arctan\left(cx^3\right)\right)\,dx=\operatorname{atan}\left(c\,x^3\right)\,\left(\frac{b\,e\,x^2}{2}+b\,d\,x\right)$$
$$+\left(\sum_{k=1}^{6}\ln\left(-\mathrm{root}\left(4096\,a^6\,c^4-1024\,a^3\,b^3\,c^3\,d^3+576\,a^2\,b^4\,c^2\,d^2\,e^2-48\,a\,b^5\,c\,d\,e^4+64\,b^6\,c^2\,d^6+b^6\,e^6,a,k\right)\,\left(\mathrm{r}\right.\right.\right.$$
$$\left.-\frac{243\,b^5\,c^9\,d^4\,e}{2}-\frac{243\,b^5\,c^9\,d^3\,e^2\,x}{4}\right)\,\mathrm{root}\left(4096\,a^6\,c^4-1024\,a^3\,b^3\,c^3\,d^3+576\,a^2\,b^4\,c^2\,d^2\,e^2\right.$$
$$\left.\left.-48\,a\,b^5\,c\,d\,e^4+64\,b^6\,c^2\,d^6+b^6\,e^6,a,k\right)\right)+a\,d\,x+\frac{a\,e\,x^2}{2}$$

```
[In] int((a + b*atan(c*x^3))*(d + e*x),x)
```

```
[Out] atan(c*x^3)*(b*d*x + (b*e*x^2)/2) + symsum(log(- root(4096*a^6*c^4 - 1024*a
^3*b^3*c^3*d^3 + 576*a^2*b^4*c^2*d^2*e^2 - 48*a*b^5*c*d*e^4 + 64*b^6*c^2*d^
6 + b^6*e^6, a, k)*(root(4096*a^6*c^4 - 1024*a^3*b^3*c^3*d^3 + 576*a^2*b^4*
c^2*d^2*e^2 - 48*a*b^5*c*d*e^4 + 64*b^6*c^2*d^6 + b^6*e^6, a, k)*(root(4096
*a^6*c^4 - 1024*a^3*b^3*c^3*d^3 + 576*a^2*b^4*c^2*d^2*e^2 - 48*a*b^5*c*d*e^
4 + 64*b^6*c^2*d^6 + b^6*e^6, a, k)*(1944*b^2*c^10*d*e - 486*b^2*c^10*e^2*x
 + 3888*root(4096*a^6*c^4 - 1024*a^3*b^3*c^3*d^3 + 576*a^2*b^4*c^2*d^2*e^2
- 48*a*b^5*c*d*e^4 + 64*b^6*c^2*d^6 + b^6*e^6, a, k)*b*c^11*d*x) - (243*b^3
*c^9*e^3)/2) - 486*b^4*c^10*d^4*x) - (243*b^5*c^9*d^4*e)/2 - (243*b^5*c^9*d
^3*e^2*x)/4)*root(4096*a^6*c^4 - 1024*a^3*b^3*c^3*d^3 + 576*a^2*b^4*c^2*d^2
*e^2 - 48*a*b^5*c*d*e^4 + 64*b^6*c^2*d^6 + b^6*e^6, a, k), k, 1, 6) + a*d*x
 + (a*e*x^2)/2
```

3.30 $\int \frac{a+b\arctan(cx^3)}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 739

$$\int \frac{a+b\arctan(cx^3)}{d+ex}\,dx = \frac{(a+b\arctan(cx^3))\log(d+ex)}{e} + \frac{bc\log\left(\frac{e\left(1-\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(-\frac{e\left(1+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e} + \frac{bc\log\left(-\frac{e\left(\sqrt[3]{-1}+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(-\frac{e\left((-1)^{2/3}+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-(-1)^{2/3}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} + \frac{bc\log\left(\frac{(-1)^{2/3}e\left(1+\sqrt[3]{-1}\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(\frac{\sqrt[3]{-1}e\left(1+(-1)^{2/3}\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-(-1)^{2/3}e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)}{2\sqrt{-c^2}e}$$

```
[Out] (a+b*arctan(c*x^3))*ln(e*x+d)/e+1/2*b*c*ln(e*(1-(-c^2)^(1/6)*x)/((-c^2)^(1/
6)*d+e))*ln(e*x+d)/e/(-c^2)^(1/2)-1/2*b*c*ln(-e*(1+(-c^2)^(1/6)*x)/((-c^2)^
(1/6)*d-e))*ln(e*x+d)/e/(-c^2)^(1/2)+1/2*b*c*ln(-e*((-1)^(1/3)+(-c^2)^(1/6)
*x)/((-c^2)^(1/6)*d-(-1)^(1/3)*e))*ln(e*x+d)/e/(-c^2)^(1/2)-1/2*b*c*ln(-e*(
(-1)^(2/3)+(-c^2)^(1/6)*x)/((-c^2)^(1/6)*d-(-1)^(2/3)*e))*ln(e*x+d)/e/(-c^2
)^(1/2)+1/2*b*c*ln((-1)^(2/3)*e*(1+(-1)^(1/3)*(-c^2)^(1/6)*x)/((-c^2)^(1/6)
*d+(-1)^(2/3)*e))*ln(e*x+d)/e/(-c^2)^(1/2)-1/2*b*c*ln((-1)^(1/3)*e*(1+(-1)^
(2/3)*(-c^2)^(1/6)*x)/((-c^2)^(1/6)*d+(-1)^(1/3)*e))*ln(e*x+d)/e/(-c^2)^(1/
2)-1/2*b*c*polylog(2,(-c^2)^(1/6)*(e*x+d)/((-c^2)^(1/6)*d-e))/e/(-c^2)^(1/2
)+1/2*b*c*polylog(2,(-c^2)^(1/6)*(e*x+d)/((-c^2)^(1/6)*d+e))/e/(-c^2)^(1/2)
+1/2*b*c*polylog(2,(-c^2)^(1/6)*(e*x+d)/((-c^2)^(1/6)*d-(-1)^(1/3)*e))/e/(-
c^2)^(1/2)-1/2*b*c*polylog(2,(-c^2)^(1/6)*(e*x+d)/((-c^2)^(1/6)*d+(-1)^(1/3
)*e))/e/(-c^2)^(1/2)-1/2*b*c*polylog(2,(-c^2)^(1/6)*(e*x+d)/((-c^2)^(1/6)*d
-(-1)^(2/3)*e))/e/(-c^2)^(1/2)+1/2*b*c*polylog(2,(-c^2)^(1/6)*(e*x+d)/((-c^
2)^(1/6)*d+(-1)^(2/3)*e))/e/(-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used

$= \{4976, 281, 209, 2463, 266, 2441, 2440, 2438\}$

$$\int \frac{a+b\arctan(cx^3)}{d+ex}\,dx = \frac{\log(d+ex)\left(a+b\arctan(cx^3)\right)}{e} - \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-(-1)^{2/3}e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)}{2\sqrt{-c^2}e} + \frac{bc\log(d+ex)\log\left(\frac{e\left(1-\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+e}\right)}{2\sqrt{-c^2}e} - \frac{bc\log(d+ex)\log\left(-\frac{e\left(\sqrt[6]{-c^2}x+1\right)}{\sqrt[6]{-c^2}d-e}\right)}{2\sqrt{-c^2}e} + \frac{bc\log(d+ex)\log\left(-\frac{e\left(\sqrt[6]{-c^2}x+\sqrt[3]{-1}\right)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e} - \frac{bc\log(d+ex)\log\left(-\frac{e\left(\sqrt[6]{-c^2}x+(-1)^{2/3}\right)}{\sqrt[6]{-c^2}d-(-1)^{2/3}e}\right)}{2\sqrt{-c^2}e} + \frac{bc\log(d+ex)\log\left(\frac{(-1)^{2/3}e\left(\sqrt[3]{-1}\sqrt[6]{-c^2}x+1\right)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)}{2\sqrt{-c^2}e} - \frac{bc\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[6]{-c^2}x+1\right)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e}$$

```
[In] Int[(a + b*ArcTan[c*x^3])/(d + e*x),x]
```

```
[Out] ((a + b*ArcTan[c*x^3])*Log[d + e*x])/e + (b*c*Log[(e*(1 - (-c^2)^(1/6)*x))/
((-c^2)^(1/6)*d + e)]*Log[d + e*x])/(2*Sqrt[-c^2]*e) - (b*c*Log[-((e*(1 + (
-c^2)^(1/6)*x))/((-c^2)^(1/6)*d - e))]*Log[d + e*x])/(2*Sqrt[-c^2]*e) + (b*
c*Log[-((e*((-1)^(1/3) + (-c^2)^(1/6)*x))/((-c^2)^(1/6)*d - (-1)^(1/3)*e))]
*Log[d + e*x])/(2*Sqrt[-c^2]*e) - (b*c*Log[-((e*((-1)^(2/3) + (-c^2)^(1/6)*
x))/((-c^2)^(1/6)*d - (-1)^(2/3)*e))]*Log[d + e*x])/(2*Sqrt[-c^2]*e) + (b*c
*Log[((-1)^(2/3)*e*(1 + (-1)^(1/3)*(-c^2)^(1/6)*x))/((-c^2)^(1/6)*d + (-1)^
(2/3)*e)]*Log[d + e*x])/(2*Sqrt[-c^2]*e) - (b*c*Log[((-1)^(1/3)*e*(1 + (-1)
^(2/3)*(-c^2)^(1/6)*x))/((-c^2)^(1/6)*d + (-1)^(1/3)*e)]*Log[d + e*x])/(2*S
qrt[-c^2]*e) - (b*c*PolyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-c^2)^(1/6)*d - e
)])/(2*Sqrt[-c^2]*e) + (b*c*PolyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-c^2)^(1/
6)*d + e)])/(2*Sqrt[-c^2]*e) + (b*c*PolyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-
c^2)^(1/6)*d - (-1)^(1/3)*e)])/(2*Sqrt[-c^2]*e) - (b*c*PolyLog[2, ((-c^2)^(
1/6)*(d + e*x))/((-c^2)^(1/6)*d + (-1)^(1/3)*e)])/(2*Sqrt[-c^2]*e) - (b*c*P
olyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-c^2)^(1/6)*d - (-1)^(2/3)*e)])/(2*Sqr
t[-c^2]*e) + (b*c*PolyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-c^2)^(1/6)*d + (-1
)^(2/3)*e)])/(2*Sqrt[-c^2]*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 4976

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*((a + b*ArcTan[c*x^n])/e), x] - Dist[b*c*(n/e), Int[x^
(n - 1)*(Log[d + e*x]/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, n
}, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(a+b\arctan(cx^3))\log(d+ex)}{e}-\frac{(3bc)\int\frac{x^2\log(d+ex)}{1+c^2x^6}\,dx}{e}\\
&= \frac{(a+b\arctan(cx^3))\log(d+ex)}{e}-\frac{(3bc)\int\left(-\frac{c^2x^2\log(d+ex)}{2\sqrt{-c^2}\left(\sqrt{-c^2}-c^2x^3\right)}-\frac{c^2x^2\log(d+ex)}{2\sqrt{-c^2}\left(\sqrt{-c^2}+c^2x^3\right)}\right)dx}{e}\\
&= \frac{(a+b\arctan(cx^3))\log(d+ex)}{e}-\frac{\left(3bc\sqrt{-c^2}\right)\int\frac{x^2\log(d+ex)}{\sqrt{-c^2}-c^2x^3}\,dx}{2e}-\frac{\left(3bc\sqrt{-c^2}\right)\int\frac{x^2\log(d+ex)}{\sqrt{-c^2}+c^2x^3}\,dx}{2e}\\
&= \frac{(a+b\arctan(cx^3))\log(d+ex)}{e}\\
&\quad-\frac{\left(3bc\sqrt{-c^2}\right)\int\left(\frac{\log(d+ex)}{3(-c^2)^{5/6}\left(1-\sqrt[6]{-c^2}x\right)}+\frac{\log(d+ex)}{3(-c^2)^{5/6}\left(-\sqrt[3]{-1}-\sqrt[6]{-c^2}x\right)}+\frac{\log(d+ex)}{3(-c^2)^{5/6}\left((-1)^{2/3}-\sqrt[6]{-c^2}x\right)}\right)d}{2e}\\
&\quad-\frac{\left(3bc\sqrt{-c^2}\right)\int\left(\frac{\log(d+ex)}{3(-c^2)^{5/6}\left(1+\sqrt[6]{-c^2}x\right)}+\frac{\log(d+ex)}{3(-c^2)^{5/6}\left(-\sqrt[3]{-1}+\sqrt[6]{-c^2}x\right)}+\frac{\log(d+ex)}{3(-c^2)^{5/6}\left((-1)^{2/3}+\sqrt[6]{-c^2}x\right)}\right)d}{2e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a+b\arctan(cx^3))\log(d+ex)}{e} - \frac{(bc)\int \frac{\log(d+ex)}{1-\sqrt[6]{-c^2}x}\,dx}{2\sqrt[3]{-c^2}e} \\
&\quad - \frac{(bc)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}-\sqrt[6]{-c^2}x}\,dx}{2\sqrt[3]{-c^2}e} - \frac{(bc)\int \frac{\log(d+ex)}{(-1)^{2/3}-\sqrt[6]{-c^2}x}\,dx}{2\sqrt[3]{-c^2}e} - \frac{(bc)\int \frac{\log(d+ex)}{1+\sqrt[6]{-c^2}x}\,dx}{2\sqrt[3]{-c^2}e} \\
&\quad - \frac{(bc)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}+\sqrt[6]{-c^2}x}\,dx}{2\sqrt[3]{-c^2}e} - \frac{(bc)\int \frac{\log(d+ex)}{(-1)^{2/3}+\sqrt[6]{-c^2}x}\,dx}{2\sqrt[3]{-c^2}e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a+b\arctan(cx^3))\log(d+ex)}{e} + \frac{bc\log\left(\frac{e\left(1-\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&\quad - \frac{bc\log\left(-\frac{e\left(1+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e} + \frac{bc\log\left(-\frac{e\left(\sqrt[3]{-1}+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&\quad - \frac{bc\log\left(-\frac{e\left((-1)^{2/3}+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-(-1)^{2/3}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&\quad + \frac{bc\log\left(\frac{(-1)^{2/3}e\left(1+\sqrt[3]{-1}\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&\quad - \frac{bc\log\left(\frac{\sqrt[3]{-1}e\left(1+(-1)^{2/3}\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} \\
&\quad - \frac{(bc)\int \frac{\log\left(\frac{e\left(1-\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+e}\right)}{d+ex}\,dx}{2\sqrt{-c^2}} - \frac{(bc)\int \frac{\log\left(\frac{e\left(-\sqrt[3]{-1}-\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{d+ex}\,dx}{2\sqrt{-c^2}} \\
&\quad - \frac{(bc)\int \frac{\log\left(\frac{e\left((-1)^{2/3}-\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)}{d+ex}\,dx}{2\sqrt{-c^2}} + \frac{(bc)\int \frac{\log\left(\frac{e\left(1+\sqrt[6]{-c^2}x\right)}{-\sqrt[6]{-c^2}d+e}\right)}{d+ex}\,dx}{2\sqrt{-c^2}} \\
&\quad + \frac{(bc)\int \frac{\log\left(\frac{e\left(-\sqrt[3]{-1}+\sqrt[6]{-c^2}x\right)}{-\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{d+ex}\,dx}{2\sqrt{-c^2}} + \frac{(bc)\int \frac{\log\left(\frac{e\left((-1)^{2/3}+\sqrt[6]{-c^2}x\right)}{-\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)}{d+ex}\,dx}{2\sqrt{-c^2}}
\end{aligned}
$$

$$= \frac{(a+b\arctan(cx^3))\log(d+ex)}{e}+\frac{bc\log\left(\frac{e\left(1-\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$-\frac{bc\log\left(-\frac{e\left(1+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e}+\frac{bc\log\left(-\frac{e\left(\sqrt[3]{-1}+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$-\frac{bc\log\left(-\frac{e\left((-1)^{2/3}+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-(-1)^{2/3}e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$+\frac{bc\log\left(\frac{(-1)^{2/3}e\left(1+\sqrt[3]{-1}\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$-\frac{bc\log\left(\frac{\sqrt[3]{-1}e\left(1+(-1)^{2/3}\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)\log(d+ex)}{2\sqrt{-c^2}e}$$

$$+\frac{(bc)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[6]{-c^2}x}{-\sqrt[6]{-c^2}d+e}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-c^2}e}$$

$$-\frac{(bc)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[6]{-c^2}x}{\sqrt[6]{-c^2}d+e}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-c^2}e}$$

$$+\frac{(bc)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[6]{-c^2}x}{-\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-c^2}e}$$

$$-\frac{(bc)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[6]{-c^2}x}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-c^2}e}$$

$$+\frac{(bc)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[6]{-c^2}x}{-\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)}{x}dx,x,d+ex\right)}{2\sqrt{-c^2}e}$$

$$-\frac{(bc)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[6]{-c^2}x}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)}{x}dx,x,d+ex\right)}{}$$

$$= \frac{(a+b\arctan(cx^3))\log(d+ex)}{e} + \frac{bc\log\left(\frac{e\left(1-\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(-\frac{e\left(1+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-e}\right)\log(d+ex)}{2\sqrt{-c^2}e} + \frac{bc\log\left(-\frac{e\left(\sqrt[3]{-1}+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(-\frac{e\left((-1)^{2/3}+\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d-(-1)^{2/3}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} + \frac{bc\log\left(\frac{(-1)^{2/3}e\left(1+\sqrt[3]{-1}\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\log\left(\frac{\sqrt[3]{-1}e\left(1+(-1)^{2/3}\sqrt[6]{-c^2}x\right)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)\log(d+ex)}{2\sqrt{-c^2}e} - \frac{bc\,\mathrm{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\mathrm{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\mathrm{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\mathrm{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e} - \frac{bc\,\mathrm{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-(-1)^{2/3}e}\right)}{2\sqrt{-c^2}e} + \frac{bc\,\mathrm{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+(-1)^{2/3}e}\right)}{2\sqrt{-c^2}e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.98 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.71

$$\int \frac{a+b\arctan(cx^3)}{d+ex}\,dx = \frac{a\log(d+ex)}{e} + \frac{b\left(2\arctan(cx^3)\log(d+ex) - i\left(\log\left(\frac{e\left(-i+\sqrt{3}-2\sqrt[3]{c}x\right)}{2\sqrt[3]{c}d+\left(-i+\sqrt{3}\right)e}\right)\log(d+ex) - \log\left(\frac{e\left(i+\sqrt{3}-2\sqrt[3]{c}x\right)}{2\sqrt[3]{c}d+\left(i+\sqrt{3}\right)e}\right)\log(d+ex\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(2*ArcTan[c*x^3]*Log[d + e*x] - I*(Log[(e*(-I + Sqr
t[3] - 2*c^(1/3)*x))/(2*c^(1/3)*d + (-I + Sqrt[3])*e)]*Log[d + e*x] - Log[(
e*(I + Sqrt[3] - 2*c^(1/3)*x))/(2*c^(1/3)*d + (I + Sqrt[3])*e)]*Log[d + e*x
] + Log[(e*(I - c^(1/3)*x))/(c^(1/3)*d + I*e)]*Log[d + e*x] - Log[-((e*(I +
 c^(1/3)*x))/(c^(1/3)*d - I*e))]*Log[d + e*x] - Log[(e*(-I + Sqrt[3] + 2*c^
(1/3)*x))/(-2*c^(1/3)*d + (-I + Sqrt[3])*e)]*Log[d + e*x] + Log[(e*(I + Sqr
t[3] + 2*c^(1/3)*x))/(-2*c^(1/3)*d + (I + Sqrt[3])*e)]*Log[d + e*x] - PolyL
og[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d - I*e)] + PolyLog[2, (c^(1/3)*(d + e*x
))/(c^(1/3)*d + I*e)] - PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d + I*e
 - Sqrt[3]*e)] + PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d + (-I + Sqrt
[3])*e)] + PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d - (I + Sqrt[3])*e)
] - PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d + (I + Sqrt[3])*e)])))/(2
*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.23

method	result
default	$\frac{a\ln(ex+d)}{e}+\frac{b\ln(ex+d)\arctan(c\,x^3)}{e}-\frac{b\,e^2\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^6c^2-6c^2d\_Z^5+15c^2d^2\_Z^4-20c^2d^3\_Z^3+15c^2d^4\_Z^2-6c^2d^5\_Z+c^2d\right.}\right.}{2c}$
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\ln(ex+d)\arctan(c\,x^3)}{e}-\frac{b\,e^2\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^6c^2-6c^2d\_Z^5+15c^2d^2\_Z^4-20c^2d^3\_Z^3+15c^2d^4\_Z^2-6c^2d^5\_Z+c^2d\right.}\right.}{2c}$
risch	$\frac{ib\ln(ex+d)\ln(-ic\,x^3+1)}{2e}-\frac{ib\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3-3\_Z^2cd+3\_Zc\,d^2-c\,d^3+e^3\,\mathrm{RootOf}\left(\_Z^2+1,\mathrm{index}=1\right)\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R}{\_R1}\right.\right.\right.}{2e}$

```
[In] int((a+b*arctan(c*x^3))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(e*x+d)/e+b*ln(e*x+d)/e*arctan(c*x^3)-1/2*b/c*e^2*sum(1/(_R1^3-3*_R1^2*
d+3*_R1*d^2-d^3)*(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1)),_
R1=RootOf(_Z^6*c^2-6*_Z^5*c^2*d+15*_Z^4*c^2*d^2-20*_Z^3*c^2*d^3+15*_Z^2*c^2
*d^4-6*_Z*c^2*d^5+c^2*d^6+e^6))
```

Fricas [F]

$$\int \frac{a+b\arctan(cx^3)}{d+ex}\,dx = \int \frac{b\arctan(cx^3)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*arctan(c*x^3) + a)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx^3)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(c*x**3))/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\arctan(cx^3)}{d+ex}\,dx = \int \frac{b\arctan(cx^3)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d),x, algorithm="maxima")
[Out] 2*b*integrate(1/2*arctan(c*x^3)/(e*x + d), x) + a*log(e*x + d)/e
```

Giac [F]

$$\int \frac{a+b\arctan(cx^3)}{d+ex}\,dx = \int \frac{b\arctan(cx^3)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan\left(cx^3\right)}{d+ex}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x^3)}{d+e\,x}\,dx$$

```
[In] int((a + b*atan(c*x^3))/(d + e*x),x)
[Out] int((a + b*atan(c*x^3))/(d + e*x), x)
```

3.31 $\int \frac{a+b\arctan(cx^3)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 906

$$\int \frac{a+b\arctan(cx^3)}{(d+ex)^2}\,dx = -\frac{bc^{2/3}de^3\arctan\left(\sqrt[3]{c}x\right)}{c^2d^6+e^6}+\frac{bc^2d^5\arctan(cx^3)}{e(c^2d^6+e^6)}-\frac{a+b\arctan(cx^3)}{e(d+ex)}+\frac{bc^{2/3}d\left(\sqrt{3}cd^3+e^3\right)\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{2(c^2d^6+e^6)}+\frac{bc^{2/3}d\left(\sqrt{3}cd^3-e^3\right)\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{2(c^2d^6+e^6)}+\frac{\sqrt{3}bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\arctan\left(\frac{1+\frac{2c^{2/3}x}{\sqrt[6]{-c^2}}}{\sqrt{3}}\right)}{2(-c^2)^{2/3}(c^2d^6+e^6)}-\frac{\sqrt{3}bc^{5/3}e\left(\sqrt{-c^2}d^3-e^3\right)\arctan\left(\frac{c^{4/3}+2(-c^2)^{5/6}x}{\sqrt{3}c^{4/3}}\right)}{2(-c^2)^{2/3}(c^2d^6+e^6)}+\frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\sqrt[6]{-c^2}-c^{2/3}x\right)}{2(-c^2)^{2/3}(c^2d^6+e^6)}-\frac{bc^{5/3}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(\sqrt[6]{-c^2}+c^{2/3}x\right)}{2(-c^2)^{2/3}(c^2d^6+e^6)}+\frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6}+\frac{bc^{5/3}d^4\log\left(1+c^{2/3}x^2\right)}{2(c^2d^6+e^6)}-\frac{bc^{2/3}d\left(cd^3-\sqrt{3}e^3\right)\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4(c^2d^6+e^6)}-\frac{bc^{2/3}d\left(cd^3+\sqrt{3}e^3\right)\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4(c^2d^6+e^6)}+\frac{bc^{5/3}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(\sqrt[3]{-c^2}-c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2\right)}{4(-c^2)^{2/3}(c^2d^6+e^6)}-\frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\sqrt[3]{-c^2}+c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2\right)}{4(-c^2)^{2/3}(c^2d^6+e^6)}-\frac{bcd^2e^2\log(1+c^2x^6)}{2(c^2d^6+e^6)}$$

```
[Out] -b*c^(2/3)*d*e^3*arctan(c^(1/3)*x)/(c^2*d^6+e^6)+b*c^2*d^5*arctan(c*x^3)/e/
(c^2*d^6+e^6)+(-a-b*arctan(c*x^3))/e/(e*x+d)+3*b*c*d^2*e^2*ln(e*x+d)/(c^2*d
^6+e^6)+1/2*b*c^(5/3)*d^4*ln(1+c^(2/3)*x^2)/(c^2*d^6+e^6)-1/2*b*c*d^2*e^2*l
n(c^2*x^6+1)/(c^2*d^6+e^6)+1/2*b*c^(2/3)*d*arctan(2*c^(1/3)*x+3^(1/2))*(-e^
```

```
3+c*d^3*3^(1/2))/(c^2*d^6+e^6)-1/2*b*c^(2/3)*d*arctan(2*c^(1/3)*x-3^(1/2))*
(e^3+c*d^3*3^(1/2))/(c^2*d^6+e^6)-1/4*b*c^(2/3)*d*ln(1+c^(2/3)*x^2-c^(1/3)*
x*3^(1/2))*(c*d^3-e^3*3^(1/2))/(c^2*d^6+e^6)-1/4*b*c^(2/3)*d*ln(1+c^(2/3)*x
^2+c^(1/3)*x*3^(1/2))*(c*d^3+e^3*3^(1/2))/(c^2*d^6+e^6)-1/2*b*c^(5/3)*e*ln(
(-c^2)^(1/6)+c^(2/3)*x)*(-e^3+d^3*(-c^2)^(1/2))/(-c^2)^(2/3)/(c^2*d^6+e^6)+
1/4*b*c^(5/3)*e*ln((-c^2)^(1/3)-c^(2/3)*(-c^2)^(1/6)*x+c^(4/3)*x^2)*(-e^3+d
^3*(-c^2)^(1/2))/(-c^2)^(2/3)/(c^2*d^6+e^6)-1/2*b*c^(5/3)*e*arctan(1/3*(c^(
4/3)+2*(-c^2)^(5/6)*x)/c^(4/3)*3^(1/2))*3^(1/2)*(-e^3+d^3*(-c^2)^(1/2))/(-c
^2)^(2/3)/(c^2*d^6+e^6)+1/2*b*c^(5/3)*e*ln((-c^2)^(1/6)-c^(2/3)*x)*(e^3+d^3
*(-c^2)^(1/2))/(-c^2)^(2/3)/(c^2*d^6+e^6)-1/4*b*c^(5/3)*e*ln((-c^2)^(1/3)+c
^(2/3)*(-c^2)^(1/6)*x+c^(4/3)*x^2)*(e^3+d^3*(-c^2)^(1/2))/(-c^2)^(2/3)/(c^2
*d^6+e^6)+1/2*b*c^(5/3)*e*arctan(1/3*(1+2*c^(2/3)*x/(-c^2)^(1/6))*3^(1/2))*
3^(1/2)*(e^3+d^3*(-c^2)^(1/2))/(-c^2)^(2/3)/(c^2*d^6+e^6)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 906, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules

used = {4980, 6857, 1890, 1430, 649, 209, 266, 648, 631, 210, 642, 1525, 298, 31, 1483}

$$\int \frac{a+b\arctan(cx^3)}{(d+ex)^2}\,dx = \frac{bc^2\arctan(cx^3)\,d^5}{e(c^2d^6+e^6)} + \frac{bc^{5/3}\log\left(c^{2/3}x^2+1\right)d^4}{2(c^2d^6+e^6)} + \frac{3bce^2\log(d+ex)d^2}{c^2d^6+e^6} - \frac{bce^2\log(c^2x^6+1)\,d^2}{2(c^2d^6+e^6)} - \frac{bc^{2/3}e^3\arctan\left(\sqrt[3]{c}x\right)d}{c^2d^6+e^6} + \frac{bc^{2/3}\left(\sqrt{3}cd^3+e^3\right)\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)d}{2(c^2d^6+e^6)} + \frac{bc^{2/3}\left(\sqrt{3}cd^3-e^3\right)\arctan\left(2\sqrt[3]{c}x+\sqrt{3}\right)d}{2(c^2d^6+e^6)} - \frac{bc^{2/3}\left(cd^3-\sqrt{3}e^3\right)\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)d}{4(c^2d^6+e^6)} - \frac{bc^{2/3}\left(cd^3+\sqrt{3}e^3\right)\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)d}{4(c^2d^6+e^6)} - \frac{a+b\arctan(cx^3)}{e(d+ex)} + \frac{\sqrt{3}bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\arctan\left(\frac{\frac{2c^{2/3}x}{\sqrt[6]{-c^2}}+1}{\sqrt{3}}\right)}{2(-c^2)^{2/3}(c^2d^6+e^6)} - \frac{\sqrt{3}bc^{5/3}e\left(\sqrt{-c^2}d^3-e^3\right)\arctan\left(\frac{c^{4/3}+2(-c^2)^{5/6}x}{\sqrt{3}c^{4/3}}\right)}{2(-c^2)^{2/3}(c^2d^6+e^6)} + \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\sqrt[6]{-c^2}-c^{2/3}x\right)}{2(-c^2)^{2/3}(c^2d^6+e^6)} - \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(c^{2/3}x+\sqrt[6]{-c^2}\right)}{2(-c^2)^{2/3}(c^2d^6+e^6)} + \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(c^{4/3}x^2-c^{2/3}\sqrt[6]{-c^2}x+\sqrt[3]{-c^2}\right)}{4(-c^2)^{2/3}(c^2d^6+e^6)} - \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(c^{4/3}x^2+c^{2/3}\sqrt[6]{-c^2}x+\sqrt[3]{-c^2}\right)}{4(-c^2)^{2/3}(c^2d^6+e^6)}$$

```
[In] Int[(a + b*ArcTan[c*x^3])/(d + e*x)^2,x]
```

```
[Out] -((b*c^(2/3)*d*e^3*ArcTan[c^(1/3)*x])/(c^2*d^6 + e^6)) + (b*c^2*d^5*ArcTan[
c*x^3])/(e*(c^2*d^6 + e^6)) - (a + b*ArcTan[c*x^3])/(e*(d + e*x)) + (b*c^(2
```

```
/3)*d*(Sqrt[3]*c*d^3 + e^3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(2*(c^2*d^6 + e^
6)) + (b*c^(2/3)*d*(Sqrt[3]*c*d^3 - e^3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(2*
(c^2*d^6 + e^6)) + (Sqrt[3]*b*c^(5/3)*e*(Sqrt[-c^2]*d^3 + e^3)*ArcTan[(1 +
(2*c^(2/3)*x)/(-c^2)^(1/6))/Sqrt[3]])/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) - (S
qrt[3]*b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3)*ArcTan[(c^(4/3) + 2*(-c^2)^(5/6)*
x)/(Sqrt[3]*c^(4/3))]])/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) + (b*c^(5/3)*e*(Sqr
t[-c^2]*d^3 + e^3)*Log[(-c^2)^(1/6) - c^(2/3)*x])/(2*(-c^2)^(2/3)*(c^2*d^6
+ e^6)) - (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3)*Log[(-c^2)^(1/6) + c^(2/3)*x]
)/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) + (3*b*c*d^2*e^2*Log[d + e*x])/(c^2*d^6
+ e^6) + (b*c^(5/3)*d^4*Log[1 + c^(2/3)*x^2])/(2*(c^2*d^6 + e^6)) - (b*c^(2
/3)*d*(c*d^3 - Sqrt[3]*e^3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(4*(c
^2*d^6 + e^6)) - (b*c^(2/3)*d*(c*d^3 + Sqrt[3]*e^3)*Log[1 + Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/(4*(c^2*d^6 + e^6)) + (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3
)*Log[(-c^2)^(1/3) - c^(2/3)*(-c^2)^(1/6)*x + c^(4/3)*x^2])/(4*(-c^2)^(2/3)
*(c^2*d^6 + e^6)) - (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 + e^3)*Log[(-c^2)^(1/3) +
c^(2/3)*(-c^2)^(1/6)*x + c^(4/3)*x^2])/(4*(-c^2)^(2/3)*(c^2*d^6 + e^6)) - (
b*c*d^2*e^2*Log[1 + c^2*x^6])/(2*(c^2*d^6 + e^6))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
 Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1525

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[-(e/2 + c*(d/(2*q))), Int[(f*
x)^m/(q - c*x^n), x], x] + Dist[e/2 - c*(d/(2*q)), Int[(f*x)^m/(q + c*x^n),
```

```
x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 4980

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_) + (e_.)*(x_))^(m_.), x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(e*(m + 1))), x] - D
ist[b*c*(n/(e*(m + 1))), Int[x^(n - 1)*((d + e*x)^(m + 1)/(1 + c^2*x^(2*n))
), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan\left(cx^3\right)}{e(d+ex)}+\frac{(3bc)\int\frac{x^2}{(d+ex)(1+c^2x^6)}\,dx}{e}\\
&= -\frac{a+b\arctan\left(cx^3\right)}{e(d+ex)}+\frac{(3bc)\int\left(\frac{d^2e^4}{(c^2d^6+e^6)(d+ex)}+\frac{(d-ex)(-e^4+c^2d^4x^2+c^2d^2e^2x^4)}{(c^2d^6+e^6)(1+c^2x^6)}\right)dx}{e}\\
&= -\frac{a+b\arctan\left(cx^3\right)}{e(d+ex)}+\frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6}+\frac{(3bc)\int\frac{(d-ex)(-e^4+c^2d^4x^2+c^2d^2e^2x^4)}{1+c^2x^6}\,dx}{e\left(c^2d^6+e^6\right)}\\
&= -\frac{a+b\arctan\left(cx^3\right)}{e(d+ex)}+\frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6}\\
&\quad+\frac{(3bc)\int\left(\frac{-de^4-c^2d^4ex^3}{1+c^2x^6}+\frac{x(e^5+c^2d^3e^2x^3)}{1+c^2x^6}+\frac{x^2(c^2d^5-c^2d^2e^3x^3)}{1+c^2x^6}\right)dx}{e\left(c^2d^6+e^6\right)}\\
&= -\frac{a+b\arctan\left(cx^3\right)}{e(d+ex)}+\frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6}+\frac{(3bc)\int\frac{-de^4-c^2d^4ex^3}{1+c^2x^6}\,dx}{e\left(c^2d^6+e^6\right)}\\
&\quad+\frac{(3bc)\int\frac{x(e^5+c^2d^3e^2x^3)}{1+c^2x^6}\,dx}{e\left(c^2d^6+e^6\right)}+\frac{(3bc)\int\frac{x^2(c^2d^5-c^2d^2e^3x^3)}{1+c^2x^6}\,dx}{e\left(c^2d^6+e^6\right)}
\end{aligned}$$

$$= -\frac{a+b\arctan(cx^3)}{e(d+ex)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} + \frac{\left(b\sqrt[3]{c}\right)\int \frac{-2c^{2/3}de^4-\left(c^2d^4e-\sqrt{3}cde^4\right)x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2e\left(c^2d^6+e^6\right)} + \frac{\left(b\sqrt[3]{c}\right)\int \frac{-2c^{2/3}de^4+\left(-c^2d^4e-\sqrt{3}cde^4\right)x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2e\left(c^2d^6+e^6\right)} + \frac{\left(b\sqrt[3]{c}\right)\int \frac{-c^{2/3}de^4+c^2d^4ex}{1+c^{2/3}x^2}\,dx}{e\left(c^2d^6+e^6\right)} + \frac{(bc)\text{Subst}\left(\int \frac{c^2d^5-c^2d^2e^3x}{1+c^2x^2}\,dx,x,x^3\right)}{e\left(c^2d^6+e^6\right)} - \frac{\left(3bc^3e\left(d^3+\frac{e^3}{\sqrt{-c^2}}\right)\right)\int \frac{x}{\sqrt{-c^2}-c^2x^3}\,dx}{2\left(c^2d^6+e^6\right)} + \frac{\left(3bce\left(c^2d^3+\sqrt{-c^2}e^3\right)\right)\int \frac{x}{\sqrt{-c^2}+c^2x^3}\,dx}{2\left(c^2d^6+e^6\right)}$$

$$= -\frac{a+b\arctan(cx^3)}{e(d+ex)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} + \frac{\left(bc^{7/3}d^4\right)\int \frac{x}{1+c^{2/3}x^2}\,dx}{c^2d^6+e^6} + \frac{\left(bc^3d^5\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{e\left(c^2d^6+e^6\right)} - \frac{\left(bc^3d^2e^2\right)\text{Subst}\left(\int \frac{x}{1+c^2x^2}\,dx,x,x^3\right)}{c^2d^6+e^6} - \frac{\left(bcde^3\right)\int \frac{1}{1+c^{2/3}x^2}\,dx}{c^2d^6+e^6} + \frac{\left(bcd\left(\sqrt{3}cd^3-e^3\right)\right)\int \frac{1}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{4\left(c^2d^6+e^6\right)} + \frac{\left(b\sqrt[3]{c}\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\right)\int \frac{1}{\sqrt[6]{-c^2}+c^{2/3}x}\,dx}{2\left(c^2d^6+e^6\right)} - \frac{\left(b\sqrt[3]{c}\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\right)\int \frac{\sqrt[6]{-c^2}+c^{2/3}x}{\sqrt[3]{-c^2}-c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2}\,dx}{2\left(c^2d^6+e^6\right)} - \frac{\left(bcd\left(\sqrt{3}cd^3+e^3\right)\right)\int \frac{1}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{4\left(c^2d^6+e^6\right)} - \frac{\left(bc^{7/3}e\left(\sqrt{-c^2}d^3+e^3\right)\right)\int \frac{1}{\sqrt[6]{-c^2}-c^{2/3}x}\,dx}{2\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} + \frac{\left(bc^{7/3}e\left(\sqrt{-c^2}d^3+e^3\right)\right)\int \frac{\sqrt[6]{-c^2}-c^{2/3}x}{\sqrt[3]{-c^2}+c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2}\,dx}{2\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} - \frac{\left(bc^{2/3}d\left(cd^3-\sqrt{3}e^3\right)\right)\int \frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{4\left(c^2d^6+e^6\right)} - \frac{\left(bc^{2/3}d\left(cd^3+\sqrt{3}e^3\right)\right)\int \frac{\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{4\left(c^2d^6+e^6\right)}$$

$$
\begin{aligned}
&= -\frac{bc^{2/3}de^3\arctan\left(\sqrt[3]{c}x\right)}{c^2d^6+e^6} + \frac{bc^2d^5\arctan\left(cx^3\right)}{e\left(c^2d^6+e^6\right)} \\
&\quad - \frac{a+b\arctan\left(cx^3\right)}{e(d+ex)} + \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\sqrt[6]{-c^2}-c^{2/3}x\right)}{2\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} \\
&\quad + \frac{b\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(\sqrt[6]{-c^2}+c^{2/3}x\right)}{2\sqrt[3]{c}\left(c^2d^6+e^6\right)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} \\
&\quad + \frac{bc^{5/3}d^4\log\left(1+c^{2/3}x^2\right)}{2\left(c^2d^6+e^6\right)} - \frac{bc^{2/3}d\left(cd^3-\sqrt{3}e^3\right)\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4\left(c^2d^6+e^6\right)} \\
&\quad - \frac{bc^{2/3}d\left(cd^3+\sqrt{3}e^3\right)\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4\left(c^2d^6+e^6\right)} - \frac{bcd^2e^2\log\left(1+c^2x^6\right)}{2\left(c^2d^6+e^6\right)} \\
&\quad - \frac{\left(b\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\right)\int \frac{-c^{2/3}\sqrt[6]{-c^2}+2c^{4/3}x}{\sqrt[3]{-c^2}-c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2}\,dx}{4\sqrt[3]{c}\left(c^2d^6+e^6\right)} \\
&\quad - \frac{\left(bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\right)\int \frac{c^{2/3}\sqrt[6]{-c^2}+2c^{4/3}x}{\sqrt[3]{-c^2}+c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2}\,dx}{4\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} \\
&\quad + \frac{\left(3bc^{7/3}e\left(\sqrt{-c^2}d^3+e^3\right)\right)\int \frac{1}{\sqrt[3]{-c^2}+c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2}\,dx}{4\sqrt{-c^2}\left(c^2d^6+e^6\right)} \\
&\quad - \frac{\left(bc^{2/3}d\left(3cd^3-\sqrt{3}e^3\right)\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{6\left(c^2d^6+e^6\right)} \\
&\quad - \frac{\left(bc^{2/3}d\left(3cd^3+\sqrt{3}e^3\right)\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{6\left(c^2d^6+e^6\right)} \\
&\quad + \frac{\left(3b\sqrt[3]{c}e\left(c^2d^3+\sqrt{-c^2}e^3\right)\right)\int \frac{1}{\sqrt[3]{-c^2}-c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2}\,dx}{4\left(c^2d^6+e^6\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc^{2/3}de^3\arctan\left(\sqrt[3]{c}x\right)}{c^2d^6+e^6} + \frac{bc^2d^5\arctan\left(cx^3\right)}{e\left(c^2d^6+e^6\right)} \\
&\quad - \frac{a+b\arctan\left(cx^3\right)}{e(d+ex)} + \frac{bc^{2/3}d\left(\sqrt{3}cd^3+e^3\right)\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{2\left(c^2d^6+e^6\right)} \\
&\quad + \frac{bc^{2/3}d\left(\sqrt{3}cd^3-e^3\right)\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{2\left(c^2d^6+e^6\right)} \\
&\quad + \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\sqrt[6]{-c^2}-c^{2/3}x\right)}{2\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} \\
&\quad + \frac{b\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(\sqrt[6]{-c^2}+c^{2/3}x\right)}{2\sqrt[3]{c}\left(c^2d^6+e^6\right)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} \\
&\quad + \frac{bc^{5/3}d^4\log\left(1+c^{2/3}x^2\right)}{2\left(c^2d^6+e^6\right)} - \frac{bc^{2/3}d\left(cd^3-\sqrt{3}e^3\right)\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4\left(c^2d^6+e^6\right)} \\
&\quad - \frac{bc^{2/3}d\left(cd^3+\sqrt{3}e^3\right)\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4\left(c^2d^6+e^6\right)} \\
&\quad - \frac{b\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(\sqrt[3]{-c^2}-c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2\right)}{4\sqrt[3]{c}\left(c^2d^6+e^6\right)} \\
&\quad - \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\sqrt[3]{-c^2}+c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2\right)}{4\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} - \frac{bcd^2e^2\log\left(1+c^2x^6\right)}{2\left(c^2d^6+e^6\right)} \\
&\quad - \frac{\left(3b\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2c^{2/3}x}{\sqrt[6]{-c^2}}\right)}{2\sqrt[3]{c}\left(c^2d^6+e^6\right)} \\
&\quad - \frac{\left(3bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2c^{2/3}x}{\sqrt[6]{-c^2}}\right)}{2\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)}
\end{aligned}
$$

$$= -\frac{bc^{2/3}de^3\arctan\left(\sqrt[3]{c}x\right)}{c^2d^6+e^6} + \frac{bc^2d^5\arctan\left(cx^3\right)}{e\left(c^2d^6+e^6\right)} - \frac{a+b\arctan\left(cx^3\right)}{e(d+ex)} + \frac{bc^{2/3}d\left(\sqrt{3}cd^3+e^3\right)\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{2\left(c^2d^6+e^6\right)} + \frac{bc^{2/3}d\left(\sqrt{3}cd^3-e^3\right)\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{2\left(c^2d^6+e^6\right)} + \frac{\sqrt{3}bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\arctan\left(\frac{1+\frac{2c^{2/3}x}{\sqrt[6]{-c^2}}}{\sqrt{3}}\right)}{2\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} + \frac{\sqrt{3}b\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\arctan\left(\frac{c^{4/3}+2\left(-c^2\right)^{5/6}x}{\sqrt{3}c^{4/3}}\right)}{2\sqrt[3]{c}\left(c^2d^6+e^6\right)} + \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\sqrt[6]{-c^2}-c^{2/3}x\right)}{2\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} + \frac{b\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(\sqrt[6]{-c^2}+c^{2/3}x\right)}{2\sqrt[3]{c}\left(c^2d^6+e^6\right)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} + \frac{bc^{5/3}d^4\log\left(1+c^{2/3}x^2\right)}{2\left(c^2d^6+e^6\right)} - \frac{bc^{2/3}d\left(cd^3-\sqrt{3}e^3\right)\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4\left(c^2d^6+e^6\right)} - \frac{bc^{2/3}d\left(cd^3+\sqrt{3}e^3\right)\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{4\left(c^2d^6+e^6\right)} - \frac{b\sqrt[3]{-c^2}e\left(\sqrt{-c^2}d^3-e^3\right)\log\left(\sqrt[3]{-c^2}-c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2\right)}{4\sqrt[3]{c}\left(c^2d^6+e^6\right)} - \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\sqrt[3]{-c^2}+c^{2/3}\sqrt[6]{-c^2}x+c^{4/3}x^2\right)}{4\left(-c^2\right)^{2/3}\left(c^2d^6+e^6\right)} - \frac{bcd^2e^2\log\left(1+c^2x^6\right)}{2\left(c^2d^6+e^6\right)}$$

Mathematica [A] (verified)

Time = 10.48 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.59

$$\int\frac{a+b\arctan\left(cx^3\right)}{(d+ex)^2}\,dx = \frac{-4a\sqrt[3]{c}(c^2d^6+e^6)-4bcd\left(c^{4/3}d^4-c^{2/3}d^2e^2+e^4\right)(d+ex)\arctan\left(\sqrt[3]{c}x\right)-4b\sqrt[3]{c}(c^2d^6+e^6)\arctan\left(cx^3\right)-}{}$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/(d + e*x)^2,x]
```

```
[Out] (-4*a*c^(1/3)*(c^2*d^6 + e^6) - 4*b*c*d*(c^(4/3)*d^4 - c^(2/3)*d^2*e^2 + e^
4)*(d + e*x)*ArcTan[c^(1/3)*x] - 4*b*c^(1/3)*(c^2*d^6 + e^6)*ArcTan[c*x^3]
```

```
- 2*b*c^(2/3)*(2*c^(5/3)*d^5 - Sqrt[3]*c^(4/3)*d^4*e + c*d^3*e^2 - c^(1/3)*
d*e^4 + Sqrt[3]*e^5)*(d + e*x)*ArcTan[Sqrt[3] - 2*c^(1/3)*x] + 2*b*c^(2/3)*
(2*c^(5/3)*d^5 + Sqrt[3]*c^(4/3)*d^4*e + c*d^3*e^2 - c^(1/3)*d*e^4 - Sqrt[3
]*e^5)*(d + e*x)*ArcTan[Sqrt[3] + 2*c^(1/3)*x] + 12*b*c^(4/3)*d^2*e^3*(d +
e*x)*Log[d + e*x] + 2*b*e*(c^2*d^4 + c^(2/3)*e^4)*(d + e*x)*Log[1 + c^(2/3)
*x^2] - b*c^(2/3)*e*(c^(4/3)*d^4 - Sqrt[3]*c*d^3*e - Sqrt[3]*c^(1/3)*d*e^3
+ e^4)*(d + e*x)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2] - b*c^(2/3)*e*(c^
(4/3)*d^4 + Sqrt[3]*c*d^3*e + Sqrt[3]*c^(1/3)*d*e^3 + e^4)*(d + e*x)*Log[1
+ Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2] - 2*b*c^(4/3)*d^2*e^3*(d + e*x)*Log[1 +
c^2*x^6])/(4*c^(1/3)*e*(c^2*d^6 + e^6)*(d + e*x))
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 862, normalized size of antiderivative = 0.95

method	result	size
default	Expression too large to display	862
parts	Expression too large to display	862

```
[In] int((a+b*arctan(c*x^3))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(e*x+d)/e+b*(-1/(e*x+d)/e*arctan(c*x^3)+3*c/e*(d^2*e^3/(c^2*d^6+e^6)*ln(
e*x+d)+(-1/12*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3)*c
^2*d^4*e-1/6*(1/c^2)^(7/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*c^2*d*e^4-1/12
*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3)*c^2*d^4*e-1/6*
(1/c^2)^(7/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*c^2*d*e^4+1/3*(1/c^2)^(4/3)
*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*3^(1/2)*c^2*e^5+1/6*ln(x^2+(1/c^2)^(1/3)
)*(1/c^2)^(2/3)*c^2*d^4*e-1/3*(1/c^2)^(7/6)*arctan(x/(1/c^2)^(1/6))*c^2*d*e
^4+1/3/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))*d^3*e^2+1/6/(1/c^2)^(1/6)*arct
an(2*x/(1/c^2)^(1/6)+3^(1/2))*d^3*e^2-1/6*(1/c^2)^(1/3)*arctan(2*x/(1/c^2)^
(1/6)+3^(1/2))*3^(1/2)*e^5+1/3*(1/c^2)^(1/2)*arctan(2*x/(1/c^2)^(1/6)+3^(1/
2))*c^2*d^5+1/6/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*d^3*e^2-1/6
*(1/c^2)^(1/3)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*3^(1/2)*e^5+1/3*(1/c^2)^(1
/2)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*c^2*d^5-1/3*(1/c^2)^(1/2)*arctan(x/(1
/c^2)^(1/6))*c^2*d^5-1/12*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/
c^2)^(1/3)*e^5-1/6*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*d^2*e^3-1/
12*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(1/3)*e^5-1/6*ln(x
^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*d^2*e^3+1/6*ln(x^2+(1/c^2)^(1/3))
*(1/c^2)^(1/3)*e^5-1/6*ln(x^2+(1/c^2)^(1/3))*d^2*e^3+1/6*(1/c^2)^(2/3)*arct
an(2*x/(1/c^2)^(1/6)+3^(1/2))*3^(1/2)*c^2*d^4*e+1/12*ln(x^2-3^(1/2)*(1/c^2)
^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(7/6)*c^2*d*e^4+1/12*ln(x^2-3^(1/2)
*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*c^2*d^3*e^2-1/6*(1/c^
2)^(2/3)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*3^(1/2)*c^2*d^4*e-1/12*ln(x^2+3^
(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(7/6)*c^2*d*e^4-1/12*l
n(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*c^2*d^3*
```

```
e^2)/(c^2*d^6+e^6)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx^3)}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx^3)}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(c*x**3))/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.51

$$\int \frac{a+b\arctan(cx^3)}{(d+ex)^2}\,dx = \frac{1}{4}\left(\left(\left(\frac{12\,d^2e^2\log(ex+d)}{c^2d^6+e^6} - \frac{\frac{4\left(c^{\frac{8}{3}}d^5-c^2d^3e^2+c^{\frac{4}{3}}de^4\right)\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}} - \frac{2\left(\sqrt{3}c^{\frac{8}{3}}d^4e+2\,c^3d^5+c^{\frac{7}{3}}d^3e^2-\sqrt{3}c^{\frac{4}{3}}e^5-c^{\frac{5}{3}}de^4\right)\arctan\left(\frac{2\,c^{\frac{2}{3}}}{\cdots}\right.}{c^2}}{\cdots}\right.\right.\right. - \frac{a}{e^2x+de}$$

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d)^2,x, algorithm="maxima")
[Out] 1/4*((12*d^2*e^2*log(e*x + d)/(c^2*d^6 + e^6) - (4*(c^(8/3)*d^5 - c^2*d^3*e
^2 + c^(4/3)*d*e^4)*arctan(c^(1/3)*x)/c^(5/3) - 2*(sqrt(3)*c^(8/3)*d^4*e +
2*c^3*d^5 + c^(7/3)*d^3*e^2 - sqrt(3)*c^(4/3)*e^5 - c^(5/3)*d*e^4)*arctan((
2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^2 + 2*(sqrt(3)*c^(8/3)*d^4*e - 2*
c^3*d^5 - c^(7/3)*d^3*e^2 - sqrt(3)*c^(4/3)*e^5 + c^(5/3)*d*e^4)*arctan((2*
```

```
c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^2 + (sqrt(3)*c^(7/3)*d^3*e^2 + c^(8
/3)*d^4*e + sqrt(3)*c^(5/3)*d*e^4 + 2*c^2*d^2*e^3 + c^(4/3)*e^5)*log(c^(2/3
)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^2 - (sqrt(3)*c^(7/3)*d^3*e^2 - c^(8/3)*d^4
*e + sqrt(3)*c^(5/3)*d*e^4 - 2*c^2*d^2*e^3 - c^(4/3)*e^5)*log(c^(2/3)*x^2 -
 sqrt(3)*c^(1/3)*x + 1)/c^2 - 2*(c^(8/3)*d^4*e - c^2*d^2*e^3 + c^(4/3)*e^5)
*log(c^(2/3)*x^2 + 1)/c^2)/(c^2*d^6*e + e^7))*c - 4*arctan(c*x^3)/(e^2*x +
d*e))*b - a/(e^2*x + d*e)
```

Giac [F]

$$\int \frac{a + b\arctan(cx^3)}{(d+ex)^2}\,dx = \int \frac{b\arctan(cx^3) + a}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 2105, normalized size of antiderivative = 2.32

$$\int \frac{a + b\arctan(cx^3)}{(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int((a + b*atan(c*x^3))/(d + e*x)^2,x)
```

```
[Out] symsum(log((729*b^6*c^14*d*e^2 + 54432*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^
6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^
4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^6*c^12*e^15*x + 729*b^6*c^14*e^3*x - 311
04*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2
*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^6
*c^14*d^7*e^8 - 243*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8
*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 +
 b^6*c^2, z, k)*b^5*c^15*d^5 + 62208*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6
+ 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*
c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^6*c^12*d*e^14 + 5832*root(64*c^2*d^6*e^6*z
^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*
e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^2*b^4*c^14*d^3*e^4 - 1944
*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d
^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^3*b
^3*c^15*d^7*e^2 + 15552*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2
*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z
^2 + b^6*c^2, z, k)^4*b^2*c^14*d^5*e^6 - 10692*root(64*c^2*d^6*e^6*z^6 + 64
*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3
```

```
 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^3*b^3*c^13*d*e^8 + 101088*root(6
4*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*
z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^5*b*c^13*d
^3*e^10 - 3888*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5
+ 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*
c^2, z, k)^5*b*c^15*d^9*e^4 - 12636*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 +
 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c
^2*d^2*e^2*z^2 + b^6*c^2, z, k)^3*b^3*c^13*e^9*x - 38880*root(64*c^2*d^6*e^
6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3
*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^6*c^14*d^6*e^9*x + 116
640*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^
2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^
5*b*c^13*d^2*e^11*x - 11664*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c
*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e
^2*z^2 + b^6*c^2, z, k)^5*b*c^15*d^8*e^5*x + 11664*root(64*c^2*d^6*e^6*z^6
+ 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6
*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^2*b^4*c^14*d^2*e^5*x - 3888*
root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^
4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)^3*b^
3*c^15*d^6*e^3*x + 38880*root(64*c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^
2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*
z^2 + b^6*c^2, z, k)^4*b^2*c^14*d^4*e^7*x - 243*root(64*c^2*d^6*e^6*z^6 + 6
4*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^4 - 16*b^3*c*e^6*z^
3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k)*b^5*c^15*d^4*e*x)/e^4)*root(64*
c^2*d^6*e^6*z^6 + 64*e^12*z^6 + 192*b*c*d^2*e^8*z^5 + 48*b^2*c^2*d^4*e^4*z^
4 - 16*b^3*c*e^6*z^3 + 12*b^4*c^2*d^2*e^2*z^2 + b^6*c^2, z, k), k, 1, 6) -
a/(d*e + e^2*x) - (b*atan(c*x^3))/(d*e + e^2*x) + (3*b*c*d^2*e^2*log(d + e*
x))/(e^6 + c^2*d^6)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
         else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
         fi;
     else #ExpnType(result) > ExpnType(optimal)
         if debug then
             print("ExpnType(result) > ExpnType(optimal)");
         fi;
         return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
     fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```